INTERNATIONAL CONFERENCE

PROBABILITY, RELIABILITY AND STOCHASTIC OPTIMIZATION

Dedicated to the 90th anniversary of V. S. Koroliuk,
80th anniversary of I. M. Kovalenko,
75th anniversary of P. S. Knopov and
75th anniversary of Yu. V. Kozachenko

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CONFERENCE MATERIALS
ORGANIZED BY
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Institute of Mathematics of the National Academy of Sciences of Ukraine
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National Technical University of Ukraine “Kiev Polytechnical Institute”
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VOLODYMYR SEMENOVYCH KOROLIUK
(TO THE 90TH ANNIVERSARY)

On August 19 there will be 90th anniversary of the outstanding Ukrainian scientist in the field of probability theory, mathematical statistics and cybernetics – Academician of NAS of Ukraine Volodymyr Semenovych Koroliuk.

V. S. Koroliuk was born in Kyiv, where he received a secondary education. Being in the military service, he graduated from the first two courses of Kharkiv University by correspondence, and since 1947 continued his studies at Taras Shechenko Kyiv University, which he graduated in 1950. Research interests of the future scientist were shaped by the influence of Academician B. V. Gnedenko.

Since 1954 he is constantly working at the Institute of Mathematics of NAS of Ukraine (former Academy of Sciences of USSR), first as a research assistant, then since 1956 – as a senior researcher, and since 1960 as the Head of the Department of probability theory and mathematical statistics. In 1963 he defended his Doctoral thesis “Asymptotical analysis in boundary problems of random walks”. Since 1966 till 1988 V. S. Koroliuk was the Deputy Director of the Institute of Mathematics of AS of USSR. In 1967 he was elected a Corresponding member and in 1976 – the Academician of the AS of USSR.

V. S. Koroliuk is one of the first scientists in Ukraine, who assessed the theoretical and practical importance of semi-Markov processes and attracted the attention of his students for their research and application. The results of these studies launched a new direction – the theory of asymptotic phase merging and averaging of random processes. They were summarized in the monographs of V. S. Koroliuk and A. F. Turbin: “Semi-Markov processes and their applications” (in Russian – 1976), “Mathematical foundations of the state lumping of large systems” (in Russian – 1978, in English – 1993) and a manual “State lumping of large systems” (in Russian – 1978).


Since 1990-th years V. S. Koroliuk continued expansion of new asymptotic methods for evolutionary systems with random perturbations. Long years of creative collaboration in the studying of phase merging of V. S. Koroliuk and N. Limnios – Professor of Technological University of Compiègne (France) – promoted in 2005 (the 80-th anniversary of the hero of the day) another monograph: V. S. Koroliuk, N. Limnios “Stochastic systems in merging phase space” of World Scientific Publishers.

The mathematical heritage of V. S. Koroliuk covers 22 monographs and about 20 textbooks, most of which are reprinted in foreign languages; about 280 scientific articles, about 50 popular science articles and editorial publications to “Encyclopedia of Cybernetics” (in Russian and Ukrainian), monographs, reference books and scientific collections.

The outstanding scientist combines fruitful scientific work, teaching and scientific-organizational activities. Since 1954 he lectured on the theory of programming, probability theory and mathematical statistics at the Taras Shevchenko Kyiv University (Faculty of mechanics and mathematics). The “Manual on probability theory and mathematical statistics” (in Russian) was published in 1978 by his editorship and was repeatedly reprinted in different languages. As a part of a group of famous experts, he was awarded by the USSR State Prize (1978) for the creation of “Encyclopedia of Cybernetics”. In addition, V. S. Koroliuk was awarded by the Glushkov Prize (1988) and Bogolyubov Prize (1995). In 1998 he was awarded by the honorary title “Honored Figure of Science and Technics of Ukraine.” In 2002 V. S. Koroliuk was awarded by the Prize of the National Academy of Sciences of Ukraine and Medal in the name of M. V. Ostrogradsky, and in 2003 – the State Prize of Ukraine in Science and Technology.

Under his supervision, 42 students defended their PhD dissertations, 14 – Doctoral dissertations.

At his age of 90 the Scientist continues active scientific, educational and organizational work, lives in creative pursuits and plans. He combines creative activities, lectures and presentations at various international conferences and in scientific centers of Italy, Spain, Holland, Germany, France, Switzerland and Sweden. He is actively involved in organizing and conducting of international conferences. He is the Editor-in-Chief of “Theory of Probability and Mathematical Statistics”, associate editor of “Ukrainian Mathematical Journal”, “Cybernetics and Systems Analysis”, “Theory of Stochastic Processes” and other journals.
Till some last years before the anniversary V. S. Koroliuk realized his dream: he applied the methods of solving the singular perturbation problem to the problems of large deviations for random evolutions.

Recent selected publications:


March 16, 2015 we celebrated 80 years since the birth of the outstanding scientist – mathematician and cybernetician – Professor, Doctor of Sciences in Physics and Mathematics, Doctor of Technical Sciences, Academician of the National Academy of Sciences of Ukraine, Laureate of the State Prizes of the USSR, UkrSSR and Ukraine Igor Mykolayovych Kovalenko.

I. M. Kovalenko was born in Kyiv on 16.03.1935. His parents Mykola Oleksandrovych Kovalenko (1904–1977) and Valeriya Volodymyrivna Yavon (1913–1997) were engineers-designers and builders. Childhood of Igor Kovalenko, including the years of World War II, was held in the village of Sloboda in Chernihiv region, where his grandparents, Volodymyr Mykhailovych Yavon and Zoya Ivanivna Yavon (Reichardt), worked as teachers.

From 1946 to 1961 Igor Kovalenko lived in Kyiv with his father. Igor Kovalenko married in 1957. With his wife Olena Markivna Kovalenko (Braga), they had been married for 55 years until her death in 2012. They have two daughters – Galyna and Yeva.

Igor Kovalenko received his primary education in a school in the village of Sloboda. Then his father, demobilized from the army, took Igor to one of the best Ukrainian secondary schools named after Ivan Franko (former Pavlo Galagan College) in Kyiv. After graduating with excellent marks (1952), Igor Kovalenko entered the Mechanics and Mathematics Faculty of Taras Shevchenko Kyiv State University. On the advice of Y. I. Gikhman (at that time assistant professor) Igor Kovalenko joined a group of mathematicians. Later, he was among the students, who were taken care of by the Theory of Probability and Algebra Department, which was headed by academician of AS of the Ussr B. V. Gnedenko (1912–1995).

In student years Igor Kovalenko began his scientific work: a graduate thesis was performed under supervision of V. S. Mykhalevych, who was a young teacher at that time. The formation of Igor Kovalenko as a mathematician was also promoted by professors of the Department L. A. Kaluzhnin, Y. I. Gikhman, O. S. Parasyuk, Y. M. Berezanskiy, senior undergraduate and graduate students A. G. Kostiuchenko, G. N. Sakovych, M. Y. Yadrenko.

The period of postgraduate studies under the supervision of B. V. Gnedenko at the Institute of Mathematics of the AS UkrSSR was very fruitful scientifically for Igor Kovalenko. Boris Volodymyrovych directed the efforts of his postgraduate students (apart from Igor Kovalenko, B. I. Grigelionis, M. V. Varoyvtskii, T. P. Marianovych, S. M. Brodie, A. A. Shakhbazov, T. I. Nasirova et al.) to the problems of queueing and reliability theories. In this research field Igor Kovalenko studied a range of queueing systems with impatient customers. The results of Igor Kovalenko formed the basis of his PhD thesis (1960). During the subsequent year and a half of work in the Institute of Mathematics (1960–1961) Igor Kovalenko discovered the so-called invariance (or insensitivity) criterion of the queueing. This result played its role in the early development of the insensitivity theory in the USSR and beyond.

Together with B. V. Gnedenko he wrote the book [1].

Along with his main teacher B. V. Gnedenko, Igor Kovalenko considers as his closed teachers the academicians V. S. Mykhalevych (1930–1994) and V. S. Koroliuk (b. 1925). Yes, V. S. Koroliuk engaged him in the small parameter method, in which he himself received great results.

The advices of academician A. N. Kolmogorov, especially his fruitful critical remarks, and of academician Yu. V. Linnik played a significant role in the formation of Igor Kovalenko as a scientist.

From his teacher Gnedenko Igor Kovalenko adopted his tendency to finding applied problems in order to solve them with the help of mathematical theory. So it was very natural for Igor that in January 1962 he moved to Moscow to work at the Institute “SNII-45” of the USSR Defense Ministry to study the challenges facing the systems reliability. There he worked for almost ten years – until mid-1971. The probabilistic model of the very complex defense system was created and investigated; as a by-product of this engineering study emerged the asymptotic method of reliability analysis of complex systems [2].

In “SNII-45” the responsible for science was the famous scientist, N. P. Buslenko. He got Igor Kovalenko involved to the development of general models of complex systems. In this field, Igor Kovalenko proposed the concept of the so-called piecewise linear aggregates (PLA) and piecewise linear Markov processes (PLMP). Elements of the PLA theory were included in the monograph [3], while PLMP formed the basis of multidimensional queueing processes in the monograph [4]. In 1964 Igor Kovalenko defended his doctoral thesis in technical sciences.
We should recall the Moscow student of Igor Kovalenko, who was distinguished by fundamental results in the theory of network services, – V. A. Ivnitskii. His latest books are [5,6].

The probabilistic combinatorics was another research field of Igor Kovalenko in the Moscow period. His second doctoral thesis (1970) was mainly devoted to the theory of random Boolean equations that had practical applications. For this thesis Igor Kovalenko received his degree of Doctor of Sciences in Physics and Mathematics.

B. V. Gnedenko, who moved to Moscow in 1960, organized a seminar on queueing and reliability theories in Mechanics and Mathematics Faculty of Moscow State University. The seminar was the real scientific school. B. V. Gnedenko recruited A. D. Solovoy, Yu. P. Belyaev, and Igor Kovalenko as co-leaders of the seminar.

By the invitation of Academician V. M. Glushkov Igor Kovalenko returned back to his native Kyiv in mid-1971. He headed the Department of Mathematical Methods of Reliability Theory of Complex Systems at the Institute of Cybernetics of the AS of UkrSSR. He still works at this position, that is for almost of 44 years. (The current name of the institute: V. M. Glushkov Institute of Cybernetics of NAS of Ukraine).

The traditional subject of the Department research – methods for reducing the variance of estimates of systems reliability – is headed by a disciple of Igor Kovalenko, a Corresponding Member of the NAS of Ukraine M. Yu. Kuznetsov. Joint monographs by Igor Kovalenko and M. Yu. Kuznetsov: [7,8].

Another worthy contribution relating mathematical reliability theory was done by an employee of the Department L. S. Stoikova [9]. She developed a method for evaluating upper and lower limits of system reliability in the case of incomplete information about the characteristics of the system.

In the Institute of Cybernetics Igor Kovalenko was also engaged in probabilistic combinatorics. In mid-1960 A. M. Kolmogorov formulated the limit distribution problem of the random Boolean determinant. Igor Kovalenko solved this problem, see [10]. Significant generalization of this result was developed primarily by A. O. Levitsytska, see [11–12]. A new approach to this problem was introduced by O. M. Oleksiychuk [13].

New tasks for himself Igor Kovalenko found during numerous international visits, starting from 1994 (Switzerland, United Kingdom, Germany, USA, Denmark, Israel, etc.). So, he found the first rigorous proof of the conjecture of D. Kendall from stochastic geometry [14], and found the estimation of the random cell area (Crofton cell) for Poisson point process [15].

Finally, let us note the works of Igor Kovalenko (mainly written together with O. V. Koba) [16–18] on the retrial queues. They obtained stability conditions for the systems, which generalized the systems investigated by L. Lakatos (Iowa State University, USA) and I. M. Maksimov (Institute for Cybernetics, Budapest, Hungary).

Let us note also the following professorial activity of Igor Kovalenko. In Taras Shevchenko National University of Kyiv Igor Kovalenko worked as a part-time professor during 17 academic years, beginning with 1971/1972. Several years he worked at Moscow Institute of Electronic Engineering, KVI RTU PVO and NTUУ “KП” (in the latter Igor Kovalenko was a Dean of the Faculty of Applied Mathematics).

He published several co-authored textbooks, among which [19–21].

Igor Kovalenko was the supervisor of 30 PhD theses. He considers 10 Doctors of Sciences to be his worthy disciples. These are the scientists: A. A. Aleksiiev, V. A. Ivnitsky, O. V. Koba, M. Yu. Kuznetsov, A. O. Levitsytska, V. I. Masol, O. M. Nakonechny, M. M. Savchuk, L. S. Stoikova, V. D. Shipak.

REFERENCES

PAVLO SOLOMONOVYCH KNOPOV

(TO THE 75TH ANNIVERSARY)

Pavlo Solomonovych Knopov is the famous and respected scientist in the field of computer science, statistical decision theory and stochastic systems optimal control. His scientific research is related to new problems of inaccurate information processing under conditions of incomplete data for the purpose of recognition, identification of object states and its control, mathematical problems of risk theory and its applications in various fields of economy and technology. His works are widely recognized by both domestic and foreign experts.

Doctor of Sciences (1986), Professor (1998), Corresponding member of the National Academy of Sciences of Ukraine (2012), Head of the Department of Mathematical Methods of Operations Research of V. M. Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine, Laureate of the State Prize of Ukraine in Science and Technology (2009) and V. M. Glushkov Prize (1997), awarded by Certificate of Honour of the Presidium of the National Academy of Sciences of Ukraine and by the Diploma of the Verkhovna Rada of Ukraine.

P. S. Knopov was born on May 21, 1940 in Kyiv. He graduated from high school number 21 in 1957. Then he entered the Mechanics and Mathematics Faculty of Kyiv State University named after T. G. Shevchenko, and graduated in 1962 with specialty “Mathematics”. The outstanding Ukrainian scientists A. V. Skorokhod, Yu. M. Berezanskyi, V. S. Mikhailovich, L. A. Kaluzhnin, V. S. Koroliuk, I. M. Kovalenko, M. Y. Yadrenko were his teachers.

The same year he started working in V. M. Glushkov Institute of Cybernetics of the NAS of Ukraine and passed successively positions from an engineer to the Head of Department. At that time on the initiative of V. S. Mikhailovich the joint laboratory of Institute of Cybernetics, Institute of Mathematics and Kyiv National Taras Shevchenko University was established under the direction of the reputable scientist in the field of statistical decision theory A. Ya. Dorogovtsev to study actual theoretical and applied problems of statistics of random processes and fields. The cooperation with A. Ya. Dorogovtsev, as well as with the prominent scientists in the field of optimization theory V. S. Mikhailovich, Yu. Ermoliev, N. Z. Shor and other scientists of the Institute of Cybernetics greatly influenced P. S. Knopov’s subsequent scientific work.

He has been working as a part-time Professor of Applied Statistics Department of the Faculty of Cybernetics, Taras Shevchenko National University of Kyiv for over 30 years.


He made talks at many international conferences, including the World Congress of Mathematicians in Berlin (1998), the European Congress of Mathematicians in Stockholm (2004), the 16th International Congress on Mathematical Programming (Lausanne 1997), the International Conference of Stochastic Programming (Columbia University, USA, 1998), International Conference on Risk, SAR and SRA – Europe Annual Conference (London, 2000) and many others.

His research interests include diverse branches of statistical theory of random processes and fields, the modern theory of stochastic optimization, forecasting and optimal control of stochastic dynamic multicomponent systems. Robust statistical methods of identification, statistical methods of unknown parameters estimation in the lack of a priori information, the theoretical foundations of optimal control of dynamic multicomponent stochastic systems, numerical methods for finding optimal estimates for complex cybernetic systems that operate in conditions of risk and uncertainty, developed by P. S. Knopov are widely used in risk estimation in various problems of nuclear energetics, economy, ecology, geology, recognition theory, reliability theory, the theory of stocks, and so on.
He received grants from the Royal Society (UK), London Mathematical Society (UK), DFG (German Foundation for Basic Research), University of Bolzano (Italy), etc.

He was the manager and executive in charge of many international projects: NATO projects (together with the University “Florida”, USA), STCU (together with the University “John Washington University”), INTAS, the company “Intel”, International Foundation SIU (Norway), scientific and applied research projects carried out by orders of the Ministry of Education of Ukraine, the Ministry of Defence of Ukraine, a number of other ministries and organizations are among them. In these projects important problems related to the risk of accidents at hazardous enterprises estimation in the presence of small statistical samples are resolved. Fundamentally new methods for reliability parameter estimation for systems based on the original fundamental results in the theory of estimation in conditions of insufficient statistical information were proposed. The important problem of identifying bottlenecks that occur in the production of computer equipment and component leading to the output of defective products was investigated in the joint project with the company “Intel” (USA). Its difficulty was caused by the situation when the very limited time period is available for making optimal decision based on extremely large volumes of observations. The proposed decision was based on developed methods of control and estimation using modern methods of remote sensing.

P. S. Knopov devotes a lot of time to scientific personnel training. A large number of doctors and candidates of sciences prepared their theses under his supervision. He has been being a member of the Specialized Scientific Council of Thesis Defense at V. M. Glushkov Institute of Cybernetics of the National Academy of Sciences of Ukraine for many years.
YURIY VASYLIOVYCH KOZACHENKO
(TO THE 75TH ANNIVERSARY)

Doctor of Sciences in Physics and Mathematics, Laureate of the State Prize of Ukraine in Science and Technology, Honored Personality of Science and Technology of Ukraine, Honored Professor of Taras Shevchenko National University of Kyiv, Honored Doctor of Uzhgorod National University, Professor Yury Vasyliovych Kozachenko is the Dear Teacher and guide in the world of science for hundreds of students and dozens of postgraduate students.

Yuriy Kozachenko was born in 1940 in Kyiv. In 1963 he graduated from Taras Shevchenko National University of Kyiv with specialty “Probability Theory and Mathematical Statistics”. His postgraduate study in the Institute of Mathematics of Academy of Sciences of the Ukrainian SSR was interrupted in 1964–1965 years by military service in cosmic forces. In 1968, in the Institute of Mathematics of the Ukrainian SSR under supervision of Mykhailo Yadrenko he defended his thesis for scientific degree “Candidate of Sciences in Physics and Mathematics” in specialty 01.01.05 – “Theory of Probability and Mathematical Statistics” titled “On the uniform convergence of stochastic integrals, series and properties of continuous random fields”.

Since 1967 he has been working at Taras Shevchenko National University of Kyiv at the Department of Probability Theory and Mathematical Statistics (since 2009 – Department of Probability Theory, Statistics and Actuarial Mathematics) of Mechanics and Mathematics Faculty. In 1974–1975 he worked at the Institute of Oil and Gas in Budeermes (Algeria).

In 1985, Yuriy Kozachenko defended his doctoral thesis on “Random processes in Orlicz spaces. Properties of trajectories, convergence of series and integrals”. During the years 1998–2003 he headed the Department of Probability Theory and Mathematical Statistics of Taras Shevchenko National University of Kyiv. His main scientific research interests relate to the study of the properties of random processes in various functional spaces, simulation and statistics of random processes, the theory of wavelet expansions of random processes. Yuriy Kozachenko is one of the founders of the theory of subGaussian and ϕ-subGaussian random processes, random processes from Orlicz spaces. He initiated a research aimed at determination of accuracy and reliability of computer simulation of random processes.


As a scientific supervisor Yuriy Kozachenko have prepared 38 candidates of sciences. Later, one of them Rostyslav Maiboroda defended doctoral thesis. In addition, for three more scientists – A. P. Yurachkivskyi, O. O. Kurchenko and I. K. Matsak – Yuriy Kozachenko was a scientific consultant when they were writing their doctoral theses. His students continue active research activities at leading universities and institutions around the world. He generously shares his deep and wide-ranging knowledge with students, postgraduate students, doctoral students, friends and colleagues.

Yuriy Kozachenko supports numerous international links with known scientists from Australia, the UK, Italy, Canada, USA, Finland, Croatia, Sweden and other countries. Many times he was invited to participate in research programs in Australia, Italy, Croatia, Finland, Sweden. Yu. Kozachenko conducted research within the framework of several international projects funded by grants from NATO, TEMPUS-TACIS programme of European Union and others. He headed many contractual themes, particularly with design bureaus of State enterprize of a special instrumentation “Arsenal” and Antonov Aeronautical Scientific-Technical Complex.

Yuriy Kozachenko won many prizes for scientific achievements. In 2005 he was awarded by the State Prize of Ukraine in Science and Technology; in 2005 he was awarded by the title of “Honorary Doctor of Uzhgorod National University”, in 2009 – by the title of “Professor Emeritus of Taras Shevchenko National University of Kyiv”, in 2010 – by the title of “Honored Personality of Science and Technology of Ukraine”, and in 2013 he won M. M. Krylov prize of the Presidium of National Academy of Sciences of Ukraine. Yuriy Kozachenko conducts active scientific, organizational, educational and social activities, he is Deputy Chairman of the Specialized Scientific Council of the Faculty of Mechanics and Mathematics, a member of the editorial boards of five academic journals, in particular, he is Deputy Editor-in-Chief of the scientific journal “Theory of Probability and Mathematical Statistics".
Students with pleasure attend normative and special courses that he inspiringly teaches at the Faculty of Mechanics and Mathematics of the Taras Shevchenko National University of Kyiv, in particular “Theory of Probability and Mathematical Statistics”, “Generalized Fourier analysis”, “Simulation of random processes”, “Theory of random processes from Orlicz spaces”, “Wavelet analysis” and others.

The main scientific works of Yuriy Kozachenko over the past four years:


Monographs, textbooks and tutorials:


ACTUARIAL AND FINANCIAL MATHEMATICS

ESTIMATES OF THE PROBABILITY OF BANKRUPTCY IN THE CASE OF "HEAVY TAILS"

A. Bilynskyi1, O. Kinash2

Classical risk theory assumes that large insurance claims and, therefore, large insurance payments are rare, with exponentially small probabilities. This scheme is called “model with small payments”. However, many situations are related to extreme events. Due to this fact, the true size of payments is more adequately represented by random variables distributed with “heavy tails”, which include Pareto type distributions. In this case, the total payments are determined by the maximum individual claim.

Note that the class of distributions with “heavy tails” is wide enough. In particular, it includes so called subexponential distributions, which were introduced by Chistyakov [1] in the context of the theory of branching processes.

Subexponential distributions and their role in risk theory are discussed, in particular, in the works of Von Bahr [2], Thorin and Wikstad [3], and Embrechts and Veraverbeke [4].

Review of the theory of subexponential distributions and applications in risk theory in sufficient detail is considered in work of Zinchenko [5].

The authors examined the estimates of the probability of bankruptcy for a number of subexponential distributions belonging to the $S$ class, which definition can be found in [5, p. 189].

REFERENCES


SOME CRITERIA FOR OPTIMAL FINANCIAL PORTFOLIO

I. Didmanidze1, T. Kokobinadze2, Z. Megrelishvili3, G. Kakhiani4

The aim of the report is the development of the recommendations for the investors to decide in which securities to invest the capital and which quantity. The following problems of optimization of financial portfolio are considered.

There are $n$ kinds of securities (government securities, municipal bonds, corporation stocks and etc.) with known mathematical expectations and with known covariance matrix. Leaving for the investor the choice of average efficiency and helping him in this case to minimize the variance, we get the problem of the minimization of the quadratic form $\sum_{i=0}^{n} b_{ij} x_{i} x_{j}$, where $b_{ij} = cov(R_{i}, R_{j})$ and $x_{i}$ represent the shares of distributed capital.

With the help of Lagrangian function the problem of finding a conditional extremum is reduced to the problem of finding of an unconditional extremum. If in the process of solution some $x_{i}$ becomes zero on negative, the appropriate securities are excluded from the portfolio and the problem should be solved again.

In the process of buying shares the investor can do the investment without risk. He should combine the risk and risk-free parts of a portfolio in order to minimize the variance at chosen level of the average efficiency. The structure of securities of the portfolio should repeat the structure of the large market of these securities. The investor can only vary the share of risk-free securities in his portfolio.

The variance of the portfolio is equal to $(1/n)$ average variance plus $(1-1/n)$ average covariance, that is why if the shares are not very strongly correlated with each other, the variance of the portfolio reduces with the growing amount of securities in the portfolio.

Every investor seeks to get an optimal portfolio of risk securities, but the share of the risk-free part of the investment he defines by the maximization of mean value of the quadratic function of the utility $K \cdot m_{R} - a_{t} \cdot K^{2} \cdot \sigma_{R}^{2}$, where $K$ is investor’s capital, $R$ is the casual efficiency at chosen structure of the portfolio, coefficients $m_{R} = M(R)$, $\sigma_{R}^{2} = D(R)$, $a_{t}$ define the investor’s tendency to risk, namely, when $a_{t}$ is small, then the tendency to risk is high.

By using approach described above, the authors created and successfully tested software simulations of the financial portfolio. The simulation was carried out on the base of the data of the time series of issuers of the international share market. The testing shows the high efficiency of the used methods and their stability towards changes of the fundamental data leading to instability in the market. Given methods are very effective in the connection with the neural network models of forecasting.

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On the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq N}, P)$ consider stochastic process in discrete time as evolution of risky asset price

$$S_n = S_0 \exp\{\{I(\theta \geq n)M_n^{(1)} + I(\theta < n)M_n^{(2)}\}, \quad n = 1, ..., N,$$

where $S_0 > 0$ is a constant, $(M_n^{(1)}, \mathcal{F}_n), M_0^{(1)} = 0$ and $(M_n^{(2)}, \mathcal{F}_n), M_0^{(2)} = 0$ are independent Gaussian martingales. $\theta$ is a random variable which takes values $1, 2, ..., N$, with known probabilities $p_i = P(\theta = i), i = 1, N$. The vector $(M_n^{(1)}, M_n^{(2)})$ is independent of $\theta$ and $I(\theta)$ is an indicator of $\theta \in \{1, N\}.

In this model we have obtained optimal in mean square hedging strategy for the European contingent claims $f(S_N)$. So, that in the special class of admissible strategies $\pi_n = (\gamma_n, \beta_n), n \in 1, N$ we minimize

$$E[|f(S_N) - X_N^\pi|^2],$$

where $X_N^\pi$ is the capital value at terminal moment $N$.

**Theorem 1.** In the model (1) of price evolution, optimal in sense (2) strategy is

$$\gamma_n = \frac{E[F_n(\Delta S_n - S_{n-1}(e^{\Delta \theta_n^{(1)}}, \sum_{i=n}^N P_i^\theta + e^{\Delta \theta_n^{(2)}} \sum_{i=1}^{n-1} Q_i^\theta - 1))/F_{n-1}^S]}{E[(\Delta S_n - S_{n-1}(e^{\Delta \theta_n^{(1)}}, \sum_{i=n}^N P_i^\theta + e^{\Delta \theta_n^{(2)}} \sum_{i=1}^{n-1} Q_i^\theta - 1))^2/F_{n-1}^S]},$$

$$\beta_n = \sum_{i=1}^n \gamma_i S_{i-1}(1 - e^{\Delta \theta_n^{(1)}}, \sum_{j=1}^N P_j^\theta - e^{\Delta \theta_n^{(2)}} \sum_{j=1}^{i-1} Q_j^\theta) = \sum_{i=1}^n S_{i-1} \Delta \gamma_i,$$

where $F_n = E[|f(S_N)/F_n^S|], F_n^S = \sigma\{S_i, i \leq n\}, P_i^\theta$ and $Q_i^\theta$ are given in [1].

**REFERENCES**


**COUPLING MOMENT FOR TIME-INHOMOGENEOUS MARKOV CHAINS**

V. V. Golomoziy¹, N. V. Kartashov²

In this presentation, we consider conditions which satisfy finiteness of the expectation of the first coupling moment for two independent discrete time-inhomogeneous Markov chains. In order to state a main result, some definitions have to be made. Let us define renewal intervals $(\theta_k^l, l \in \{0, 1\}$ for a Markov chain $(\chi_t)$:

$$\theta_0^l = \inf\{t \geq 0: \chi_t = 0\}, \theta_n^l = \inf\{t > \theta_{n-1}^l: \chi_t = 0\}, n > 1,$$

and probabilities $g_{t}^{n,l} = P(\theta_k^l = n | r_{k-1}^l = t), l = 1, 2, n \geq 0$ We agree that $g_{0}^{0,l} = P(\theta_k^l = 0 | r_{k-1}^l = t) = 0$, where $r_{k}^m = \sum_{k=0}^m \theta_k^l$. Variables $\theta_k^l, k \geq 1$ are understood as a renewal step and $\theta_0^l$ is understood as a delay.

We say that $T > 0$ is a coupling (or simultaneous hitting) moment if $T = \min\{t > 0: \exists m,n: t = r_{m}^n = \tau_{l}^2\}$.

Our goal is to find conditions which guarantee that $T < \infty$ a.s. and $E[T] < \infty$. We denote as $u_{n}^{(t,l)}$ a renewal sequence for the process $r_t^l$:

$$u_{0}^{(t,l)} = 1, u_{n}^{(t,l)} = \sum_{k=0}^n u_{k}^{(t,l)} g_{n-k}^{t+k,l}.$$

**Theorem 1.** Assume that in notations introduced before:

1) The set of random variables $\theta(t)$ is uniformly integrable (in other words, the family of distributions $g_{n}^{(t,l)}$ is uniformly integrable).

2) There exists a constant $\gamma > 0$ and positive integer $n_0 > 0$ such that for all $t, l$ and $n \geq n_0: u_{n}^{(t,l)} \geq \gamma$.

Then the coupling moment is integrable: $E[T] < \infty$.

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GENERALIZATION OF DOOB-MEYER DECOMPOSITION THEOREM AND ITS APPLICATIONS

N. S. Gonchar

In report we present results concerning optional decomposition of supermartingale relative to convex set of equivalent measures. These results generalize Doob-Meyer decomposition theorem for supermartingale [1, 2].

Theorem 1. Let \{f_t, \mathcal{F}_t\}_{t=0}^\infty be cadlag supermartingale relative to convex set of equivalent measures \(M\) being defined on measurable space \(\{\Omega, \mathcal{F}\}\) and let there exist a real number \(0 < \lambda_0 < \infty\), a measure \(\tilde{P} \in M\) such that \(E^P f_{t_0} < E^\tilde{P} f_{t_0}\).

If

\[
\sup_{P \in M} \sup_{t \in [0, d]} |f_t| = F_d < \infty, \quad 0 \leq d < \infty,
\]

then there exist \(\mathcal{F}_t\)-adapted increasing process \(g = \{g_t\}_{t=0}^\infty\), \(g_0 = 0\), and a right continuous martingale \(\{M_t, \mathcal{F}_t\}_{t=0}^\infty\) relative to convex set of equivalent measures \(M\) such that

\[
f_t = M_t - g_t.
\]

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STATISTICAL MODEL OF THE STOCK MARKET ORDER QUEUES

A. I. Kakoichenko

The price modeling problem is usually considered as a time series analysis problem. But this approach is not effective for high frequency trading applications, because it misses the state of the order book and information about transactional flows.

In recent years, with the growth of electronic exchanges and algorithmic trading, the availability of high frequency data on the transactional flows opens new opportunities for creating financial market models for short-term forecasting and high frequency trading algorithms development. In this paper we develop a model for markets with continuous double auction.

Following the seminal work by Avellaneda and Stoikov [1] the flow of transactions of placing new orders is a Poisson flow. Korolev in his work [2] considers the time spans between two transactions as gamma-distributed. But the analysis of transactional flows, made in this work, proved, that this assumption contradicts the reality.

The state of the market with continuous double auction can be considered as states of order books of buy and sell orders respectively. Order book is a queue of orders with equal direction, where new orders with opposite direction could process these orders or could be placed to the queue of orders with their direction. The process of order book changing is unambiguously described by streams of transactions. There are three most popular types of transactions: add order, remove order and modify order. The proposed model based on the range of frequencies gives the opportunity to generate events and modify the initial state to build a prediction of the orders books states. Application of this model to historical data showed wide prospect of this approach.

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SIMULATION OF FINANCIAL SERIES WITH DESIRED MULTIFRACTAL PROPERTIES

L. Kirichenko1, A. Khabachova, A. Storozhenko

Lately numerous studies have identified a number of specific features of financial series: peakedness, heavy-tailed distributions, property of self-similarity, long-term dependence, conditional volatility et. In 1996, B. Mandelbrot proposed multifractal model describing the dynamics of financial time series MMAR (Multifractal Model of Asset Returns) [1]. It is based on modeling of fractional Brownian motion in multifractal time by operation of subordination.

In the general case operation of subordination (random substitution of time ) can be represented in the form

\[
Z(t) = Y(T(t)), \quad t > 0,
\]

where \(T(t)\) is nonnegative nondecreasing stochastic process called subordinator. \(Y(t)\) is stochastic process, which is independent of \(T(t)\).

In [1] it is proved that if a stochastic process \(X(t)\) is process of subordination \(X(t) = B_H(\theta(t))\), where \(B_H(t)\) is fractional Brownian motion with Hurst parameter \(H\) and \(\theta(t)\) is subordinators, which is the distribution function of the cumulative measure defined on the interval \([0, T]\), then \(X(t)\) is the multifractal process with multifractal spectrum

\[
f_X(\alpha) = f_\theta(\frac{\alpha}{H}),
\]

where \(f_\theta(\alpha)\) is multifractal spectrum of the process \(\theta(t)\).
In [2] it is suggested the method of building a stochastic binomial cascade with weights having a beta-distribution. The proposed algorithm allows to generate a realizations of the desired values of the Hurst exponent \( H \) and the multifractal spectrum \( f(\alpha) \). Thus we can simulate the financial time series using the cascade realizations with the required fractal characteristics.

In this work we have investigated various financial time series: stock prices, indices, currency pairs, which showed obvious multifractal properties. The proposed approach allows to model the realizations with the desired multifractal characteristics.

References


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THE RATE OF CONVERGENCE OF OPTION PRICES FOR THE DISCRETIZATION OF GEOMETRIC ORNSTEIN-UHLENBECK PROCESS

Yu. S. Mishura

We take the martingale central limit theorem that was established, together with the rate of convergence, by Liptser and Shiryaev ([1]), and adapt it to the multiplicative scheme of financial markets with discrete time that converge to the standard Black-Scholes model. To improve the rate of convergence, we suppose that the increments are independent and identically distributed (but without binomial or similar restrictions on the distribution). Under additional assumptions, in particular under the assumption that absolutely continuous component of the distribution is nonzero, we apply asymptotical expansions of distribution function and establish that the rate of convergence is \( O(n^{-1/2}) \). The results are contained in [2–4].

References


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THE RATE OF CONVERGENCE TO THE NORMAL LAW IN TERMS OF PSEUDOMOMENTS

Yu. S. Mishura1, Y. Munchak2, P. Slyusarchuk3

Yu. Studnyev in [1] received estimate of the rate of convergence in terms of pseudomoments. We get the same rate of convergence avoiding the Cramer’s condition. Instead, we impose boundedness of the truncated pseudomoments \( \nu_n^{(2)} \) for some \( m \geq 3 \).

\[
\nu_n^{(2)}(m) = \int_{|x| > \sigma \sqrt{n}} |x|^m |dH(x)|.
\]

Theorem 1. Let the following conditions hold:

(i) The characteristic function is integrable, \( \int_{\mathbb{R}} |f(t)| dt = A < \infty \);

(ii) Pseudomoments up to order \( m \) inclusive equal zero and truncated pseudomoments are bounded: \( \mu_k = 0 \) for some \( m \geq 3 \), and \( \nu_n(m) = \max \{ \nu_n^{(1)}(m), \nu_n^{(2)}(m) \} < \frac{1}{2} e^{-\frac{1}{2}} \).

Then for all \( n \geq 2 \)

\[
\sup_{x \in \mathbb{R}} |\Phi_n(x) - \Phi(x)| \leq 2C_1 \frac{\nu_n^{(1)}(m)}{n^{-\frac{1}{2}}} + 2C_2 \frac{\nu_n^{(2)}(m)}{n^{-\frac{1}{4}}} + \frac{\sigma A}{\pi} b n - 1 + \nu_n(m) \frac{4 e^2}{\pi} \frac{1}{n},
\]

References


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We take the martingale central limit theorem that was established, together with the rate of convergence, by Liptser and Shiryaev ([1]), and adapt it to the multiplicative scheme of financial markets with discrete time that converge to the standard Black-Scholes model. To improve the rate of convergence, we suppose that the increments are independent and identically distributed (but without binomial or similar restrictions on the distribution). Under additional assumptions, in particular under the assumption that absolutely continuous component of the distribution is nonzero, we apply asymptotical expansions of distribution function and establish that the rate of convergence is \( O(n^{-1/2}) \). The results are contained in [2–4].

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THE RATE OF CONVERGENCE TO THE NORMAL LAW IN TERMS OF PSEUDOMOMENTS

Yu. S. Mishura1, Y. Munchak2, P. Slyusarchuk3

Yu. Studnyev in [1] received estimate of the rate of convergence in terms of pseudomoments. We get the same rate of convergence avoiding the Cramer’s condition. Instead, we impose boundedness of the truncated pseudomoments and integrability of the characteristic function. Let \( \{\xi_n, n \geq 1\} \) be a sequence of independent identically distributed random variables with \( E\xi_i = 0, D\xi_i = \sigma^2 \in (0, \infty) \), distribution function \( F(x) \) and characteristic function \( f(t) \), \( \Phi_n(x) \), \( x \in \mathbb{R} \) be the distribution function of a random variable \( S_n = (\sigma \sqrt{n})^{-1} (\xi_1 + \xi_2 + \ldots + \xi_n) \) and \( \Phi(x) \), \( x \in \mathbb{R} \) be the distribution function of the standard normal law. We assume that for some \( m \geq 3 \) there exist the pseudomoments \( \mu_k = \int_{\mathbb{R}} x^k d\Phi(x) \), \( (k = 3, \ldots, m, m \in N) \), where \( \Phi(x) = \Phi(x) - \Phi(0) \). Denote the values that will be called the truncated pseudomoments: “truncation from above” \( \nu_n^{(1)}(m) = \int_{|x| \leq \sigma \sqrt{n}} |x|^m |d\Phi(x)| \) and “truncation from below” \( \nu_n^{(2)}(m) = \int_{|x| > \sigma \sqrt{n}} |x|^m |d\Phi(x)| \).

Theorem 1. Let the following conditions hold:

(i) The characteristic function is integrable, \( \int_{\mathbb{R}} |f(t)| dt = A < \infty \);

(ii) Pseudomoments up to order \( m \) inclusive equal zero and truncated pseudomoments are bounded: \( \mu_k = 0 \) for some \( m \geq 3 \), and

\[
\nu_n(m) = \max \{ \nu_n^{(1)}(m), \nu_n^{(2)}(m) \} < \frac{1}{2} e^{-\frac{1}{2}}.
\]

Then for all \( n \geq 2 \)

\[
\sup_{x \in \mathbb{R}} |\Phi_n(x) - \Phi(x)| \leq 2C_1 \frac{\nu_n^{(1)}(m)}{n^{-\frac{1}{2}}} + 2C_2 \frac{\nu_n^{(2)}(m)}{n^{-\frac{1}{4}}} + \frac{\sigma A}{\pi} b n - 1 + \nu_n(m) \frac{4 e^2}{\pi} \frac{1}{n},
\]
where $C_{m}^{(1)} = \frac{12m+1}{2\pi(m+1)} + C_{m+1}^{(1)}$, $C_{m}^{(2)} = 2C_{m-1}^{(1)}$, $b = \exp\left\{-\frac{a^2}{24\pi^2(2+\pi)^2}\right\} < 1$.

Note that assumption (i) implies the existence of density for the random variable $S_n$. We get the corresponding conditions of convergence of densities.

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EXPONENTIAL BOUND FOR THE INFINITE-HORIZON RUIN PROBABILITY IN A RISK MODEL WITH A VARIABLE PREMIUM INTENSITY AND RISKY INVESTMENTS

Yu. S. Mishura1, O. Yu. Ragulina2

We consider a generalization of the classical risk model when the premium intensity depends on the current surplus of the insurance company (see [1]). All surplus is invested in a risky asset, the price of which follows a geometric Brownian motion. Our main aim is to show that if the premium intensity grows rapidly with increasing surplus, then an exponential bound for the infinite-horizon ruin probability holds under certain conditions in spite of the fact that all surplus is invested in the risky asset in contrast to the results of [2, 3]. To this end, we allow the surplus process to explode and investigate the question concerning the probability of its explosion between claim arrivals in detail. In particular, we consider the case when the premium intensity is a quadratic function of the current surplus.

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CONVERGENCE OF STOPPING TIMES IN JUMP-DIFFUSION MODELS

G. M. Shevchenko

We consider a sequence of stochastic differential equations in $\mathbb{R}^d$

$$X^n(t) = X^n(0) + \int_0^t a^n(s, X^n(s))ds + \int_0^t b^n(s, X^n(s))dW(s) + \int_0^t \int_{\mathbb{R}^m} c^n(s, X^n(s), \theta)\nu(d\theta, ds),$$

driven by a Wiener process $W$ and by a homogeneous Poisson random measure $\nu$ with finite intensity. Assuming the pointwise convergence $a^n \to a^0$, $b^n \to b^0$, $c^n \to c^0$ as $n \to \infty$, we prove the convergence in probability $X^n \xrightarrow{P} X^0$, $n \to \infty$. Further, defining stopping times as

$$\tau^n = \inf\{t \geq 0 : \varphi^n(t, X^n(t)) \leq 0\}, n \geq 0,$$

we show that $\tau^n \xrightarrow{P} \tau^0$, $n \to \infty$, under certain assumptions about the convergence $\varphi^n \to \varphi^0$.

We discuss the applications of our results to optimal stopping problems, in particular, in financial mathematics.

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ON CONSTRUCTION OF DISCRETE TIME SHADOW PRICE

L. Stettner1, T. Rogala2

In the case of market with proportional transaction costs (bid and ask prices) one consider a price which is between bid and ask price such that the optimal value of reward functional, which measures expected utility from consumption is the same as in the market with transaction costs. Such price is called shadow price. We shall present a method of construction of shadow price in discrete time based on a so called weak shadow price, which corresponds to illiquid market. This price in fact is a random variable depending on current investor position as well on as current bid and
ask prices such that optimal value of reward functional for it is the same as in the market with transaction costs. Such price can be constructed recursively. With the use of weak shadow price we construct a shadow price. The talk is based on paper [1] as well as PhD thesis of Tomasz Rogala.

References


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Weak limits of multivariate geometric random sums

I. V. Zolotukhin

Geometric summation arises naturally in the financial field. Portfolios of financial instruments such as stocks can change price over time resulting in a time series of random vectors. Geometric stable laws are the weak limits of normed and centered geometric random sums of i.i.d. random vectors. But more useful and realistic for portfolio modeling would be study distributions with different tail behavior of components. To provide that behavior of tails, we study limit distributions of multivariate geometric random sums in the form

\[
S = (S_1, \ldots, S_k) = \left( \sum_{j=1}^{M_1} X_1^{(j)}, \ldots, \sum_{j=1}^{M_k} X_k^{(j)} \right),
\]

where \( M_l \) (\( l = 1, \ldots, k \)) will be defined below, \( X_l^{(j)} \) are independent random variables identically distributed for each \( l \) with known characteristic function \( E^{\exp(it_1 X_l)} = \varphi_l(t_1) \), and \( M_l \) and \( X_l^{(j)} \) are independent.

The vector \( M = (M_1, \ldots, M_k) \) is introduced by the following way. Let \( \mathcal{E} = \{ \epsilon \} \) is a set of \( k \)-dimensional indices \( \epsilon = (\epsilon_1, \ldots, \epsilon_k) \) and each component of \( \epsilon_l \) is 0 or 1; \( \mathcal{E}_l \) is a set of \( k \)-dimensional indices for which \( \epsilon_l = 1 \); \( \mathcal{N}_l \) are independent geometrically distributed random variables with parameters \( p_\epsilon \). By definition, put the value \( M_l = \min \{ \mathcal{N}_l \} \).

We define the GMSGs (Generalized Marginally Strictly Geometric Stable) law as the distribution of vector \( V = (Z_1^{1/\alpha_1}Y_1, Z_2^{1/\alpha_2}Y_2, \ldots, Z_k^{1/\alpha_k}Y_k) \), where \( Y_l \) are independent random variables with strictly stable distributions with characteristic functions \( g_l(\theta_l) \) and parameters \( \alpha_l, \eta_l, \beta_l \); \( Z = (Z_1, \ldots, Z_k) \) is independent from \( Y_1, \ldots, Y_k \) random vector with the Marshall-Olkin multivariate exponential distribution. Apropos these laws can be recursively restored by distributions of their univariate components.

Now let \( p_\epsilon = \lambda_\epsilon p \). Assume that \( \varphi_l(p^{1/\alpha_l} \theta_l) = 1 + p \ln g_l(\theta_l) + o(p) \) as \( p \to 0 \). We have proved that the vector \( \left( p^{1/\alpha_1} \sum_{j=1}^{M_1} X_1^{(1)}, \ldots, p^{1/\alpha_k} \sum_{j=1}^{M_k} X_k^{(k)} \right) \) converges weakly to \( V \) as \( p \to 0 \), where \( V \) has the GMSGs-distribution.

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Analytical and numerical approach framework for the optimal reinsurance policy for the normal losses

V. P. Zubchenko

Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)\) be the probability space which satisfies standard assumptions. Let \( X \) be a non-negative random variable defined on this probability space with cumulative distribution function \( F_X(x) = P(X \leq x), x \geq 0, 0 < EX < \infty \). In what follows we use the notations introduced in [1].

Reinsurance leads to the partition of \( X \) to the following two parts. The ceded loss function \( f(X) \) represents the portion of the loss that is ceded to the reinsurer, \( 0 \leq f(X) \leq X \). The retained loss function \( R_l(X) = X - f(X) \) is the loss retained by the insurer.

We consider the optimal reinsurance problem for the normal losses. To obtain the optimal reinsurance scheme we use analytical and numerical approach frameworks.

References


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**APPROXIMATION OF WIENER PROCESS BY WIENER INTEGRALS WITH RESPECT TO FRACTIONAL BROWNIAN MOTION**

O. L. Banna

**Definition 1.** Fractional Brownian Motion (FBM) with Hurst index $H \in (0, 1)$ is a Gaussian process $\{B^H_t, t \geq 0\}$ with zero mean and covariance function $E B^H_t B^H_s = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$.

We study the case when Hurst index satisfies $H \in (\frac{1}{2}, 1)$. It is shown in [2] that the FBM $\{B^H_t, t \in [0, T]\}$ can be represented as $B^H_t = \int_0^t z(t, s) dW_s$, where $\{W_t, t \in [0, T]\}$ is a Wiener process,

$$z(t, s) = (H - \frac{1}{2}) \cdot c_H s^{1/2-H} \int_s^t u^{H-1/2} (u-s)^{H-3/2} du,$$

is a kernel, $c_H = \left( \frac{2H \Gamma(\frac{1}{2} - H)}{\Gamma(H + \frac{1}{2}) \cdot \Gamma(2 - 2H)} \right)^{1/2}$, is a constant and $\Gamma(x)$, $x > 0$ is Gamma function. The best uniform approximation of Wiener process in the space $L_\infty([0, T]; L_2(\Omega))$ by integrals of the form $\int_0^t f(s) dB^H_s$, where $\{B^H_t, t \in [0, T]\}$ is fractional Brownian motion, $f$ is a function $f(s) = k \cdot s^\alpha$, $k > 0$, $s \in [0, T]$, $\alpha = H - 1/2$, $H$ is Hurst index of fractional Brownian motion, is established [1]. New inequalities for the Gamma function are obtained as a byproduct: for any $\alpha \in (0, \frac{1}{2})$

$$\frac{(\Gamma(1 - \alpha))^3}{\Gamma(1 - 2\alpha)} < \frac{2(2\alpha + 1) \Gamma(2\alpha)}{3(4\alpha + 1) \Gamma(3\alpha)}.$$

**References**


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**TWO-DIMENSIONAL PROBABILITY DISTRIBUTIONS OF THE JESSEN-WINTNER TYPE**

N. V. Cherchuk1, G. M. Torbin2

Well known Jessen-Wintner theorem states that almost surely convergent series of independent discretely distributed random variables is of pure Lebesgue type. On the other hand in one-dimensional case there are many families of random variables which are of pure type, but the Jessen-Wintner theorem cannot be applied to prove the purity of their distributions. Firstly this phenomenon was observed by G. Torbin for the family of random variables with independent $Q$ and $Q^*$-digits: in such a case the random variable could be represented as a convergent series of discretely distributed random variables but they are, generally speaking, not independent. Many other families of random variables of such a type (random variables of the Jessen-Wintner type) were studied by S. Albeverio, M. Pratsiovytyi, G. Torbin and others. Random variables of such a type are measurable functions of sequence of independent discretely distributed random variables. Many interesting subclasses of such functions are generated by different symbolic F-representations $\Delta_F^{x_1, x_2, ..., x_n}$ of real numbers and corresponding measurable mappings $\varphi(x_1, x_2, ..., x_n) = \xi := \Delta_F^{x_1, x_2, ..., x_n}$. Conditions on $F$ under which the distribution of $\xi$ is of pure type were studied by M. Pratsiovytyi and G. Torbin. Multidimensional case are essentially less investigated. It is worth to mention research activity in this direction, which has been done by S. Albeverio, V. Koshmanenko, V. Koval, M. Pratsiovytyi, O. Shkolnyi, G. Torbin (see, e.g., [1]). During the talk we shall discuss problems, which are similar to the above mentioned ones, for two-dimensional case and demonstrate new phenomena which are absent in one-dimensional case.

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functions defined in terms of continued fractions

A. S. Chuikov

Let $x = [a_1, a_2, \ldots, a_n, \ldots]$ be the continued fraction expansion of $x \in (0; 1)$ and let $y = \Delta_{a_1a_2\ldots}^L$ be the expansion of $y \in (0; 1)$ by the Lüroth-type alternating series [2]. Consider the function

$$f([a_1, a_2, \ldots, a_n, \ldots]) = \Delta_{a_1a_2\ldots}^L.$$ 

We prove that $f(x)$ is correctly defined and monotonically increasing function. We also examine the function

$$\sigma(x) = \sigma([a_1, a_2, \ldots, a_n, \ldots]) = [a_1 + a_2, a_3, \ldots, a_n, \ldots].$$ 

It can be given by formula

$$\sigma(x) = \frac{1 - xa_1(x)}{a_1(x) + x(1 - a_1^2(x))}.$$ 

We prove that $\sigma(x) = \frac{\omega(x)}{a_1(x)\omega(x) + 1}$, where $\omega(x)$ is a one-sided shift operator

$$\omega(x) = \omega([a_1, a_2, \ldots, a_n, \ldots]) = [a_2, a_3, \ldots, a_{n+1}, \ldots].$$

Lemma 1. Function $\sigma(x)$ is correctly defined, decreasing, continuous and concave on each interval $\left(\frac{1}{n+1}, \frac{1}{n}\right)$ for any integer $n \geq 2$.

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Backward stochastic differential equations driven by multifractional brownian motion

M. Dozzi

Backward stochastic differential equations (BSDEs) serve in particular as models for trading strategies on financial markets and for recursive utilities in consumption processes. The theory is now well developed for BSDEs driven by brownian motion [1]. More recently, BSDEs driven by fractional brownian motions ([2], [3] e.g.) and BSDEs with rough drivers [4] have been studied.

This talk treats of BSDEs driven by multifractional brownian motions (mbm), i.e. fractional brownian motions whose Hurst parameter varies with time, which allows for driving trajectories with time-varying degree of oscillation. The stochastic integrals in these BSDEs are understood in the divergence sense and Itô formulas for mbm are applied to solve explicitly the linear BSDE by means of an associated partial differential equation. This is joint work with H. Knani, Sousse University, Tunisia [5].

References


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On new approach to the study of fractal properties of probability measures with independent $x - Q_\infty$-digits

Irina Garko

The report is devoted to the development of DP-approach to research the fractal properties of spectra and minimal dimensional supports of probability measures from family containing the probability measures with independent digits of $Q_\infty$-representation, $Q_\infty$-representation, Luroth expansions and their alternating modifications as special cases.

The main approach to the study of probability measures with independent $x - Q_\infty$-digits which are represented in this talk, is that for some fixed stochastic vector $Q_\infty$ and a fixed real number $x \in [0, 1]$ propose the consider the bijection

$$\varphi \left( \Delta_{a_1(z)a_2(z)\ldots a_k(z)\ldots}^Q \right) = \Delta_{x-Q_\infty}^{x-Q_\infty}_{a_1(z)a_2(z)\ldots a_k(z)\ldots}.$$
and study conditions, when \( \varphi \) preserves the Lebesgue measure and Hausdorff-Besicovitch dimension on the unit interval.

It should be noted that the properties of such bijection depend essentially on the chosen \( x \).

To investigate the DP-properties of the above bijection \( \varphi \) we must study the problem which is connected to the faithfulness of the family of coverings which are connected with the above mentioned expansions.

**References**


**MIXED GAUSSIAN PROCESSES: A FILTERING APPROACH**

M. Kleptsyna\(^1\), P. Chigansky\(^2\), C. Cai\(^3\)

We present a new approach to analysis of mixed processes

\[ X_t = B_t + G_t, \quad t \in [0, T], \]

where \( B_t \) is a Brownian motion and \( G_t \) is an independent centered Gaussian process. We obtain a new canonical innovation representation of \( X \), using linear filtering theory. When the kernel

\[ K(s, t) = \frac{\partial^2}{\partial s \partial t} \mathbb{E} G_s G_t, \quad s \neq t. \]

has a weak singularity on the diagonal, our results generalize the classical innovation formulas beyond the square integrable setting. For kernels with stronger singularity, our approach is applicable to processes with additional “fractional” structure, including the mixed fractional Brownian motion from mathematical finance. We show how previously known measure equivalence relations and semimartingale properties follow from our canonical representation in a unified way, and complement them with new formulas for Radon-Nikodym densities.

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**LIMIT THEOREMS FOR MULTIFRACTAL PRODUCTS OF STATIONARY PROCESSES**

N. N. Leonenko

Multifractal and monofractal models have been used in many applications in hydrodynamic turbulence, finance, computer network traffic, etc. (see, for example, [3]). There are many ways to construct random multifractal models ranging from simple binomial cascades to measures generated by branching processes and the compound Poisson process ([1]–[3]).

We investigate the properties of multifractal products of geometric Gaussian processes with possible long-range dependence and geometric Ornstein-Uhlenbeck processes driven by Lévy motion and their finite and infinite superpositions. We present the general conditions for the \( L_q \) convergence of cumulutive processes to the limiting processes and investigate their \( q \)-th order moments and Rényi functions, which are nonlinear, hence displaying the multifractality of the processes as constructed. We also establish the corresponding scenarios for the limiting processes, such as log-normal, log-gamma, log-tempered stable or log-normal tempered stable scenarios.

This is joint work with D. Denisov (Manchester University, UK).

**References**


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ON DP-APPROACH TO FRACTAL PROPERTIES OF RANDOM VARIABLES WITH INDEPENDENT IDENTICALLY DISTRIBUTED GLS-SYMBOLS

M. L. Lupain

The talk is devoted to the development of methods for the calculations of the Hausdorff dimension of spectra of probability measures.

Let \( x = \Delta_{\alpha_1(x)}\alpha_2(x)\ldots \alpha_n(x)\ldots \alpha_j \in N \) be a GLS-expansion of \( x \), which is a generalization of Lüroth expansion and of \( Q_\infty \)-expansion.

Let us recall that a bijection \( \psi \) is said to be transformation preserving the Hausdorff dimension on the unit interval is \( \dim_H(E) = \dim_H(\psi(E)), \forall E \subset (0,1) \).

Main attention will be paid to fractal properties of spectra \( S_\xi \) of random variables \( \xi = \Delta_{\alpha_1(x)}\alpha_2(x)\ldots \alpha_n(x)\ldots \alpha_j \) which were introduce by I. Garko to study \( x - Q_\infty \) measures [2].

To this end, we consider transformation \( \Delta_{\alpha_1(x)}\alpha_2(x)\ldots \alpha_n(x)\ldots \alpha_j \) which maps digits of GLS-expansion to the corresponding \( Q_\infty \)-expansion (in such a case the matrix \( Q_\infty^{GLS} \) coincides with \( Q_\infty \)). We develop the methods which were introduce by I. Garko to study \( x - Q_\infty \) measures [2].

**Theorem 1.** Let \( \psi = \Delta_{\alpha_1(x)}\alpha_2(x)\ldots \alpha_n(x)\ldots \alpha_j \) be a random variable with independent identically distributed \( Q_\infty \)-digits. Then \( \psi(S_\xi) = S_\psi \) and \( \dim_H(S_\xi) = \dim_H(S_\psi) \).

**Theorem 2.** Let \( \psi = \sum_{i=1}^{\infty} (\sum_{j=1}^{\infty} \psi_j) \) be a random variable with independent identically distributed \( Q_\infty \)-digits. Then the Hausdorff dimension of \( S_\xi \) can be calculated as follows: \( \dim_H(S_\xi) = \sup \{ x : \sum_{i \in \xi} q_i > 1 \} \).

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**ASYMPTOTIC PROPERTIES OF THE MEAN-TYPE INTEGRAL FUNCTIONALS OF FRACTIONAL BROWNIAN SHEETS**

V. I. Makogin

We investigate the weak convergence of integral functional of \( d \)-dimensional anisotropic self-similar random fields with \( N \) parameters to the continuous local time in case of its existence.

A real-valued fractional Brownian sheet with Hurst index \( H = (H_1,\ldots,H_N) \in (0,1)^N \) is the centered Gaussian random field \( B_H = \{ B_H(t), t \in \mathbb{R}_+^N \} \) with a covariance function

\[
E(B_H(t)B_H(s)) = 2^{-N} \prod_{i=1}^{N} \left( |t_i|^{2H_i} + |s_i|^{2H_i} - |t_i - s_i|^{2H_i} \right), \quad t, s \in \mathbb{R}_+^N.
\]

**Theorem 1.** Let \( \{ B^{H,d}(s), s \in \mathbb{R}_+^N \} \) be a \( d \)-dimensional fractional Brownian sheet with Hurst index \( H = (H,\ldots,H) \in \mathbb{R}_+^N, H \in (0,1) \). Let \( V : \mathbb{R}^d \to \mathbb{R} \) be a bounded Borel function. Denote \( J_{H,d}(V, T) := \mathbb{E} \int_{[0,T]} V(B^{H,d}(s))ds, \) where \( T \in \mathbb{R}_+^N \).

(i) If \( d = \frac{1}{H} > 0 \) and \( V := \mathbb{E} \int_{\mathbb{R}^d} V(x)||x||^{-d}dx < +\infty \), then

\[
J_{H,d}(V, T) = \frac{V}{(\log(T_1\cdots T_N))^{N-1}} \rightarrow \frac{V}{2\pi^{d/2}H (N-1)!} \left( \frac{d}{2} - \frac{1}{2H} \right), \quad \text{as} \ \min \{ T_i, i = 1,N \} \to \infty.
\]

(ii) If \( d = \frac{1}{H} < 0 \) and \( V := \mathbb{E} \int_{\mathbb{R}^d} V(x)dx < +\infty \), then

\[
J_{H,d}(V, T) = \frac{V}{(\log(T_1\cdots T_N))^{N}} \rightarrow \frac{V}{(2\pi)^{d/2}N!}, \quad \text{as} \ \min \{ T_i, i = 1,N \} \to \infty.
\]

(iii) If \( d = \frac{1}{H} < 0 \) and \( V < +\infty \), then

\[
J_{H,d}(V, T) = \frac{V}{(T_1\cdots T_N)^{1-d}} \rightarrow \frac{V}{\left( \frac{1}{1-Hd} \right)^N}, \quad \text{as} \ \min \{ T_i, i = 1,N \} \to \infty.
\]

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NUMERICAL CONSTRUCTION OF DRIFT PARAMETER ESTIMATOR FOR THE MIXED FRACTIONAL BROWNIAN MOTION

Yu. S. Mishura¹, I. G. Voronov²

Consider the problem of drift parameter estimation for the mixed fractional Brownian motion in the case when observed stochastic process is constructed as follows:

\[ X(t) = \theta t + \sigma_1 B^{H_1}(t) + \sigma_2 B^{H_2}(t), \quad t \in [0, T], \]

where \( B^{H_1} \) and \( B^{H_2} \) are two independent fractional Brownian motions with Hurst indices \( H_1 \) and \( H_2 \), such that \( \frac{1}{2} \leq H_1 < H_2 < 1 \).

In this presentation, we propose to consider the algorithm which enables to use constructed in [1] theory in order to get estimator for the \( \theta \) numerically. Moreover the algorithm works just as well even in the case when the following conditions on the parameters \( H_1 \) and \( H_2 \): \( H_1 > \frac{1}{2} \), \( H_2 - H_1 > \frac{1}{2} \) are broken. The mentioned conditions are sufficient for the existence and consistence of the drift parameter estimator (Theorems 2.3, 2.4 in [1]).

In the specific case for \( H_1 = \frac{1}{2} \) the algorithm may also be used successfully. In this case the construction is based on research in [2].

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FRACTAL PROPERTIES OF PROBABILITY MEASURES GENERATED BY RANDOM Q∞-EXPANSIONS AND RELATED PROBLEMS

R. Nikiforov

We establish several new probabilistic, fractal and number theoretical phenomena connected with the \( Q_\infty \)-expansion which is generated by iterated function systems (IFS) consisting of infinite similitudes with positive ratios \( q_i \) such that \( \sum_{i=1}^{\infty} q_i = 1 \). We show that the system of cylinders of this expansion is, generally speaking, not faithful, i.e., to determine the Hausdorff dimension of a set from the unit interval one is not restricted to consider only coverings consisting of the above mentioned cylinders. We prove sufficient conditions for the non-faithfulness of the family of \( Q_\infty \)-cylinders. On the other hand, sufficient conditions for the faithfulness of such covering systems are also found.

Based on results related to fine fractal properties of probability measures with independent \( Q_\infty \)-digits, we study the set \( L(Q_\infty) \) of \( Q_\infty \)-essentially non-normal numbers, i.e., real numbers \( x \) such that the asymptotic frequency \( \nu_i(x) \) of the digit \( i \) in the \( Q_\infty \)-expansion of \( x \) does not exist for all \( Q_\infty \)-digit \( i \in \{0, 1, 2, 3, \ldots \} \). We have shown in particular that this set is of full Hausdorff dimension.

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SYSTEMS OF ENCODING OF REAL NUMBERS WITH INFINITE ALPHABET AND PROBABILITY MEASURES WITH COMPLICATED LOCAL TOPOLOGICAL AND FRACNTAL STRUCTURE

M. V. Pratsiovtyyi

We consider systems of encoding of fractional part of real number with infinite alphabet \( A \) coinciding with \( N \) or \( \mathbb{Z}_0 \). Systems with zero, extra-zero, and non-zero redundancy are among them. We are interested in infinite-symbol encodings generated by two-symbol encodings mainly.

We study properties of probability measures with complicated local structure (fractal properties of spectrum and support, everywhere dense sets of peculiarities of probability distribution function).

Let \( \Delta_f^{a_1a_2\ldots} \) be a two-symbol continuous \( f \)-encoding of fractional part of real number \( x \) with alphabet \( A_2 = \{0, 1\} \) and extra-zero redundancy, and let

\[ \overline{X}^{a_1a_2\ldots} = \Delta_f^{1\overline{a_1}1\overline{a_2}1\ldots} \]

be its re-encoding in terms of infinite alphabet \( A \equiv \mathbb{Z}_0 = \{0, 1, 2, \ldots \} \).

In the talk, we study local and global spectral topological, metric, and fractal properties of random variables \( X = \overline{X}^{\tau_1\tau_2\ldots} \) and \( Y = \overline{Y}^{\eta_1\eta_2\ldots} \), where \( (\tau_n) \) is a sequence of independent random variables, \( (\eta_n) \) is a sequence of random variables with Markov dependence.
We also consider probability distributions of $\mathcal{F}$-digits $\theta_n$ of random variable $Z = \sum f \theta_1 \theta_2 \ldots \theta_n$ with given distribution for various systems of encoding $f$.

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SPECTRAL AND FRACTAL PROPERTIES OF SINGULAR PROBABILITY DISTRIBUTION FUNCTIONS OF MINKOWSKI TYPE

M. V. Pratsiovytyi1, T. M. Isaieva2

We study properties of strictly increasing singular a distributions functions belonging to two one-parameter families:

(1) \[ G_k(x) = G_{\mu_k}(\{0; a_1, a_2, \ldots, a_n, \ldots\}) = (1 - \mu)^{a_1-1} - (1 - \mu)^{a_2-1} \mu^{a_2} + \ldots + (1 - \mu)^{a_k-1} - (1 - \mu)^{a_k-1} \mu^{a_k} + \ldots + (1 - \mu)^{a_k-1} \mu^{a_k} \mu^{a_k+1} \ldots + a_{k+1} = \sum_{k=1}^{\infty} A_k (1 - \mu^{a_k}) \],

where $A_k = (1 - \mu)^{a_1+1} \mu^{a_1+1} \ldots a_{k+1}$ and $G_k(0) = 0, G_k(1) = 1$.

(2) $F_k(x)$ is a distribution function of random variable $\xi = [0; \eta_1, \eta_2, \ldots, \eta_k, \ldots]$ whose terms of regular continued fraction representation $\eta_k$ are independent identically distributed random variables taking the values $1, 2, \ldots, k, \ldots$ with probabilities $(1 - q), (1 - q)q, \ldots, (1 - q)^{k-1}, \ldots$; where $\mu$ and $q$ are fixed parameters belonging to interval $(0; 1)$.

These functions are two different generalizations of classic singular Minkowski function [1] whose analytical expression is given by Salem [2]:

\[ p(x) = p([0; a_1, a_2, \ldots, a_n, \ldots]) = 2^{1-a_1} - 2^{1-a_1-a_2} + 2^{1-a_1-a_2-a_3} + \ldots + (1)^{n+1} \mu^{1-a_1-\ldots-a_n} + \ldots, \quad a_n \in N. \]

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THE PROPERTIES OF THE DISTRIBUTION OF THE RANDOM VARIABLE SUCH THAT $L$-SYMBOLS ARE RANDOM VARIABLES THAT FORM A MARKOV CHAIN

M. V. Pratsiovytyi1, Yu. V. Khvorostina2

Let us consider the random variable $r.v.$ 
\[ \theta = \frac{1}{\theta_1} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1}{\theta_1(\theta_1+1) \ldots \theta_{n-1}(\theta_{n-1}+1)} \theta_n \equiv \Delta \theta_{12} \ldots \theta_n \]

that has the infinite $L$-representation [1]. The elements $\theta_n$ are random variables taking the values of positive integers form a Markov chain. The Markov chain has the initial probabilities $(p_1, p_2, \ldots, p_n, \ldots)$ and transition matrix $|p_{ik}|$. We learn the Lebesgue structure, have the distribution function $F_\theta(x)$, study topological, metric and fractal properties of the spectrum for the r.v. $\theta$.

Theorem 1. The distribution of r.v. $\theta$ has atoms if and only if there exists a sequence of positive integers $(a_1, a_2, \ldots, a_k, \ldots)$, such that $p_{a_1} \prod_{k=1}^{\infty} p_{a_k a_{k+1}} > 0$.

Theorem 2. If all elements of matrix $|p_{ik}|$ are equal to 1 or 0, then the distribution of r.v. $\theta$ is a pure discrete with a single atom in each cylinder of the first rank.

Theorem 3. If the elements of the matrix $|p_{ik}|$ are not equal to 1 and matrix $|p_{ik}|$ has at least one 0, then r.v. $\theta$ has a singularly distribution of Cantor type.
Theorem 4. If $p_i > 0$, $p_{i(i+1)} > 0$ and $p_i + p_{i(i+1)} = 1 \forall i \in \mathbb{N}$, then the spectrum of the distribution of the r.v. $\theta$ is an anomalously fractal set.

In the report we offer results of studying of properties of the distribution of r.v. $\theta$ such that $\theta_1, \theta_3, \theta_5, \ldots$ are independent and $\theta_2, \theta_4, \theta_6, \ldots$ are elements of Markov chain.

REFERENCES


ON PROBABILISTIC APPROACH TO GDP TRANSFORMATIONS

S. O. Rudnytsky¹, G. M. Torbin²

It is well known that methods of fractal geometry are extremely useful in the theory of singularly continuous probability measures, because such measures are usually supported by fractals (see, e.g., [2] and references therein). On the other hand probabilistic methods are among the most powerful tools to study fractals. During the last decade the theory of transformations preserving the Hausdorff-Besicovitch dimension (DP-transformations) is shown to be very useful for the calculation of Hausdorff-Besicovitch dimension of fractals with a non-regular local structure (see, e.g., [1]). Probabilistic approach to DP-transformations was developed in papers by S. Albeverio, M. Ibraham, M. Lebid, M. Pratsiovytyi, G. Torbin and others.

In the talk we shall introduce and discuss properties of generalized DP-transformations. We shall paid a special attention to a relatively stable subgroup of GDP-transformations consisting of transformations preserving triviality and non-triviality of the Hausdorff measure.
NEGA-\(\hat{Q}\)-REPRESENTATION OF REAL NUMBERS

S. O. Serbenyuk

Let \(\hat{Q} = ||q_{i,j}||\) be a fixed matrix, where \(i = 0, m_j, m_j \in N^0 = N \cup \{0, \infty\}, j = 1, 2, \ldots,\) such that:
1. \(\mathbb{R} \ni q_{i,j} > 0;\)
2. \(\forall j \in \mathbb{N}: \sum q_{i,j} = 1;\)
3. \(\forall (i_j), i_j \in \mathbb{N} \cup \{0\}: \prod_{j=1}^{\infty} q_{i_j,j} = 0.\)

Definition 1. A representation of \(x \in [0;1]\) by the expansion
\[
x = a_{i_1}(x,1) + \sum_{k=2}^{\infty} a_{i_k}(x,k) \prod_{j=1}^{k-1} q_{i_j,j},
\]
where \(a_{i_k,k} = \sum_{i=0}^{k-1} q_{i,k} \neq 0\) for \(i_k \neq 0\) and \(a_{0,k} = 0,\) is the \(\hat{Q}\)-expansion of \(x \in [0;1],\) and it is denoted by \(x = \Delta_{i_1(x)i_2(x)\ldots}^{\hat{Q}}\) \([1]\). This notation is \(\hat{Q}\)-representation of \(x.\)

Definition 2. The representation \(\Delta_{i_1i_2\ldots i_k}^{\hat{Q}}\) such that
\[
x = \Delta_{i_1i_2i_3\ldots i_k\ldots}^{\hat{Q}} \equiv \Delta_{i_1(x)}^{\hat{Q}}[m_2-i_2(x)]i_3(x)[m_4-i_4(x)]\ldots \equiv a_{i_1}(x,1) + \sum_{k=2}^{\infty} \tilde{a}_{i_k}(x,k) \prod_{j=1}^{k-1} \tilde{q}_{i_j,j},
\]
where \(\tilde{a}_{i_k}(x,k) = a_{i_k-i_k(x),k}, \tilde{p}_{i_k}(x) = p_{m_k-i_k(x),k}\) for an even number \(k\) and \(\tilde{a}_{i_k}(x,k) = a_{i_k(x),k}, \tilde{p}_{i_k}(x) = p_{i_k(x),k}\) for an odd number \(k,\) is called nega-\(\hat{Q}\)-representation of a number \(x \in [0;1].\)

The talk is devoted to formulation of a foundation of metric theory of real numbers nega-\(\hat{Q}\)-representation.

References

ON SINGULAR DISTRIBUTION OF RANDOM VARIABLES 
WITH INDEPENDENT SYMBOLS OF F-EXPANSION

Lilia Sinichyk

Let \(x = \Delta_{i_1(x)i_2(x)\ldots i_n(x)}^{F}\) be a generalized \(F\)-expansion of real numbers over an alphabet \(A.\) In such a general case we fix only two simple assumptions on the family of corresponding cylinders:
1) interior parts of cylinders of the same rank do not intersect;
2) \(|\Delta_{i_1i_2\ldots i_n}| \rightarrow (n \rightarrow \infty).\)

Let \(\xi_1(x), \xi_2(x), \ldots, \xi_n(x)\) be a sequence of independent random variables and \(\xi = \Delta_{i_1(x)i_2(x)\ldots i_n(x)}^{F}\) be a random variable with independent symbols of \(F\)-expansion. \(P(\xi = i_0) = p_{i_0}k.\)

Theorem 1. If there exist a symbol \(i_0 \in A,\) such that:
1) \(\sum_{k=1}^{\infty} p_{i_0}k = +\infty,\)
2) for any number \(x\) from interval \([0,1]\) and for any \(n \in \mathbb{N}\) there exists a sequence \(c_n = c_n(i_0) :\)
\[
|\Delta_{i_1(x)i_2(x)\ldots i_n(x)}^{F} | \leq c_n(i_0) \quad \text{and} \quad \sum_{n=1}^{\infty} c_n < +\infty,
\]
then
1) \(\lambda\)-almost all \(x \in [0,1]\) contain symbol \(i_0\) only finitely many times;
2) the probability measure \(\mu_{i_0}\) is singular with respect to Lebesgue measure.
RATIONAL $Q_2$-REPRESENTATION OF REAL NUMBERS

S. V. Skrypnyk

We consider the rational $Q_2$-representation of the fractional part of a real number generalizing the classic binary representation. We reject a hypothesis that criterion of rationality for rational $Q_2$-representation of number is completely analogous to known criterion of rationality for binary representation.

Let $q_0$ be a fixed real number belonging to the interval $(0, 1)$, $q_1 = 1 - q_0$, $\beta_i = iq_{i-1}$, $i \in A = \{0, 1\}$, i.e., $\beta_0 = 0$, $\beta_1 = q_0$.

For any $x \in [0, 1]$, there exists a sequence $(a_n)$, $a_n \in A$, such that

$$x = \beta_{a_1} + \sum_{k=2}^{\infty} \left( \beta_{a_k} \prod_{j=1}^{k-1} q_{a_j} \right) = \Delta_{a_1a_2...a_n}^Q, \quad a_n \in A.$$

Expression is called the $Q_2$-expansion of real number $x$, and its formal notation $x = \Delta_{a_1a_2...a_n}^Q$ is called the $Q_2$-representation.

The $Q_2$-representation of real numbers is called rational, if $q_0$ is a rational number.

Lemma 1. If the rational $Q_2$-representation of $x$ is periodic, then this number is rational.

Theorem 2. If the number $q_0 = \frac{p}{s}$ is an irreducible fraction, then rational number $\frac{1}{s - p}$ has a non-periodic $Q_2$-representation.

This means that the criterion of rationality of number in terms of binary representation cannot be transferred to $Q_2$-representation of real numbers.

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FAITHFULNESS OF FINE PACKING SYSTEMS WITH RESPECT TO PACKING DIMENSION CALCULATION

O. V. Slutskyi

The report is devoted to finding conditions for the fine packing systems faithfulness with respect to packing dimension calculation. The packing dimension $\dim_p$ [2] is fractal dimension in some sense dual to the Hausdorff dimension $\dim_H$. Let us fix some family $\Phi$ of open balls from a metric space $M$.

Definition 1. If

$$\forall x \in M, \forall r > 0 \exists B \in \Phi : |B| \leq r \text{ and } x \in B,$$

then $\Phi$ is called “a fine packings system” (or FPS).

Definition 2. Suppose that some FPS $\Phi$ satisfies the following condition:

$$\forall E \subset M : \dim_{P(\text{unco})}(E, \Phi) = \dim_{P(\text{unco})}(E).$$

Then $\Phi$ is called “faithful for packing dimension with uncentered balls calculation”.

The $Q$-expansion of real numbers is a generalization of $s$-expansion and $Q$-expansion and was described, for example, in [1].

Theorem 3. Let $Q$ be the matrix $||q_{ik}||$, where $i \in \mathbb{N}$, $k \in \{0, 1, \ldots, N_k - 1\}$. If the condition

$$\lim_{i \to \infty} \frac{\ln q_{ik}}{\ln(q_{i1}q_{i2}...q_{i(k-1)})} = 0$$

holds for every sequence $(i_k)$ then FPS $\Phi$ of the respective expansion cylinders is faithful for packing dimension calculation.
Let $\Phi$ be a net and let $H^\alpha(\cdot, \Phi)$ be the corresponding net Hausdorff measure [1], which is said to be comparable if the ratio $\frac{H^\alpha(\cdot, \Phi)}{H^0(\cdot, \Phi)}$ is uniformly bounded. A net $\Phi$ is said to be faithful (non-faithful) for the Hausdorff dimension calculation on $[0,1]$ if
\[ \dim_H(E, \Phi) = \dim_H(E), \quad \forall E \subseteq [0,1] \]
(resp. $\exists E \subseteq [0,1] : \dim_H(E, \Phi) \neq \dim_H(E)$).

It is clear that any comparable net-family $\Phi$ is faithful. Initial examples of non-faithful nets appeared firstly in two-dimensional case as a result of active studies of self-affine sets during the last decade of XX century. The family of cylinders of the classical continued fraction expansion can probably be considered as the first (and rather unexpected) example of non-faithful one-dimensional net-family of coverings ([3]). Let us recall ([2]) that a transformation $F$ is said to be DP on the unit interval if $F$ preserves the Hausdorff dimension of any subset of $[0,1]$.

During the talk we present several new results on non-comparable net measures which lead to new families of cylinders. We shall also discuss the problem of stability of comparability and faithfulness under DP-transformations. Applications of results to the study of fine fractal properties of singularly continuous probability measures will also be discussed.

**References**


**NON-COMPARABLE HAUSDORFF MEASURES, DP-TRANSFORMATIONS AND FINE PROPERTIES OF PROBABILITY DISTRIBUTIONS**

G. M. Torbin

Let $\Phi$ be a net and let $H^\alpha(\cdot, \Phi)$ be the corresponding net Hausdorff measure [1], which is said to be comparable if the ratio $\frac{H^\alpha(\cdot, \Phi)}{H^0(\cdot, \Phi)}$ is uniformly bounded. A net $\Phi$ is said to be faithful (non-faithful) for the Hausdorff dimension calculation on $[0,1]$ if
\[ \dim_H(E, \Phi) = \dim_H(E), \quad \forall E \subseteq [0,1] \]
(resp. $\exists E \subseteq [0,1] : \dim_H(E, \Phi) \neq \dim_H(E)$).

It is clear that any comparable net-family $\Phi$ is faithful. Initial examples of non-faithful nets appeared firstly in two-dimensional case as a result of active studies of self-affine sets during the last decade of XX century. The family of cylinders of the classical continued fraction expansion can probably be considered as the first (and rather unexpected) example of non-faithful one-dimensional net-family of coverings ([3]). Let us recall ([2]) that a transformation $F$ is said to be DP on the unit interval if $F$ preserves the Hausdorff dimension of any subset of $[0,1]$.

During the talk we present several new results on non-comparable net measures which lead to new families of cylinders. We shall also discuss the problem of stability of comparability and faithfulness under DP-transformations. Applications of results to the study of fine fractal properties of singularly continuous probability measures will also be discussed.

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**PROBABILITY DISTRIBUTIONS RELATED TO CONTINUOUS NOWHERE DIFFERENTIABLE FUNCTIONS OF TAKAGI TYPE**

N. A. Vasylenko

Takagi function $T$ is defined by
\[ T(x) = \sum_{k=0}^{\infty} \frac{\varphi_k(x)}{2^k}, \text{ where } \varphi_k(x) = \inf_{m \in \mathbb{Z}} |x - m|. \]

Function $T(x)$ was introduced by T. Takagi in 1903 [1]. It is a continuous non-differentiable function on the unit interval. We construct [3] two-parametric generalization of Takagi function:
\[ T^*(x) = T^*(\Delta_{\alpha_1, \alpha_2, \ldots, \alpha_k} \ldots) = \sum_{n=0}^{\infty} \varphi^n_n(x) \prod_{k=1}^{n} q_{\alpha_k}, \]
where
\[ \varphi^n_0(x) = \begin{cases} x \text{ if } \alpha_1(x) = 0, \\ \frac{q_0}{q_1} (1 - x) \text{ if } \alpha_1(x) = 1; \end{cases} \]
\[ \varphi^n_n(x) = \begin{cases} L^n(x) \text{ if } \alpha_{n+1}(x) = 0, \\ \frac{q_0}{q_1} (1 - L^n(x)) \text{ if } \alpha_{n+1}(x) = 1; \end{cases} \]
where $L^n(x) = L^n(\Delta_{\alpha_1, \alpha_2, \ldots, \alpha_{n+1}} \ldots) = \Delta_{\alpha_1, \alpha_2, \ldots, \alpha_{n+1}} \ldots$ is the $Q_2$-expansion of $x$ [2, P. 87].

We study the Lebesgue structure, topological and metric properties of random variables defined by function $T^*(x)$.

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INFORMATION SECURITY

RANDOMIZED STREAM CIPHERS WITH ENHANCED SECURITY BASED ON NONLINEAR RANDOM CODING

A. N. Alekseychuk, S. V. Gryshakov

A generic class of randomized stream ciphers based on joint employment of encryption, linear error-correction coding, and dedicated (linear random) coding is proposed and studied in [1,2]. Further investigation of these ciphers showed [3] that their computational security significantly depends on the properties of certain components and can be considerably less than their designers claim.

We propose another framework for design of randomized stream ciphers with enhanced security. The key attribute of this framework is using of nonlinear bijective transformations or keyless hash functions for random coding. We show that proposed ciphers are provably secure against attacks described in [1,2] regardless of the cryptographic properties of underlined keystream generator (at the same time the ciphertext length exceeds the plaintext length no more than two times). In addition, we investigate the security of the proposed ciphers against a more powerful class of chosen initialization vector attacks and show that it is based on the hardness of solving some system of nonlinear Boolean equations.

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ALGEBRAIC DEGENERATE APPROXIMATIONS OF BOOLEAN FUNCTIONS FOR KEY RECOVERY ATTACKS ON STREAM CIPHERS

A. N. Alekseychuk, S. N. Konushok, A. Y. Storozhuk

A general framework for chosen initialization vector statistical analysis of stream ciphers [1,2] is developed. We propose a key recovery attack based on approximations of a Boolean function (depended on the cipher) by algebraic degenerate functions (i.e., the functions which are linear equivalent to functions of fewer variables). In contrast to the method of probabilistic neutral bits from [1,2], our technique allows to use a much wider class of approximations. This fact significantly increases the efficiency of the considered attacks. We also offer a systematic method for finding such kind of approximations based on the improved Levin’s algorithm [3].

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SIDE CHANNEL CUBE ATTACK ON CIPHER HALKA

O. M. Fal, A. Shevchenko

Cube attacks consist of two stages. The aim of first stage is to obtain a set of linear polynomials which depend on unknown key bits. On the second stage the simple system of linear equations is established and can be resolved. For the rest of key bits the brute force attack is applied.

A bit of an intermediate state in ciphers can be represented as a multivariate polynomial \( f(X) \) (where \( X = V \cup K \)) of public variables and key variables. In the preprocessing phase the adversary randomly chooses \( I_1 \), a product of the selected public variables. Then \( f(X) \) can be represented as \( f(X) = t_1 p_{S(I)} + q_1(X) \). \( I \), called a cube, is a set of indexes of public variables. \( t_1 \) is a maxterm. \( p_{S(I)} \) is called a superpoly of \( I \).
Assume all public variables not in \( t_I \) are 0. For an assignment \( a \) to the public variables in \( t_I \), \( f(X) \) becomes \( f_a(K) \). The sum of \( f_a(K) \) over \( GF(2) \) for all assignments is exactly \( p_{S(I)} \).

Let in the Hamming weight (HW) leakage model an attacker deduces the value of HW of targeted byte. Then LSB \( y_0 \) of it as \( f(X) \) can be utilized in cube attacks.

The result of current note is obtaining 52 linear polynomials on 52 out of 80 key bits for lightweight block cipher Halka [1].

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**POLYNOMIAL EQUIVALENCE OF KNOWN-PLAINTEXT ATTACKS ON SYMMETRIC AND ENDOMORPHIC CIPHERS**

A. V. Fesenko

Recently, much attention is given to research common mathematical models of cryptographic systems, which are algebraic structures – heterogeneous algebras. In fact the first algebraic model of a symmetric cipher has been proposed by C. Shannon. An sk-algebra is an algebra \( A = (K, X, Y, f) \) with a set of carriers \( (K, X, Y) \), where \( K, X \) and \( Y \) are non-empty finite sets, and one algebraic operation \( f: K \times X \to Y \) that satisfies the following conditions:

(i) \( f: K \times X \to Y \) is surjective function;
(ii) for any \( k \in K \) and \( x_1, x_2 \in X \) if \( x_1 \neq x_2 \), then \( f(k, x_1) \neq f(k, x_2) \).

An algebraic model of deterministic (finite) cipher is an sk-algebra \( A = (K, X, Y, f) \), where \( K, X \) and \( Y \) are sets of secret keys, plaintexts and ciphertexts correspondingly. An particular case of sk-algebra \( A = (K, X, Y, f) \), when \( X = Y \), is called esk-algebra \( B = (K, X, f) \) what corresponds to an algebraic model of endomorphic cipher.

Usually attacks on symmetric ciphers can be classified based on what type of information the attacker has available and one of the basic type is known-plaintext attack. Formally cipher strength to this type of attack is determined by the complexity of the solutions of the following problem.

**Problem 1.** Given sk-algebra \( A = (K, X, Y, f) \) and two arbitrary values \( x \in X \) and \( y \in Y \) such, that exists \( k \in K \), for which \( f(k, x) = y \) holds, find (all) unknown value \( k \).

**Theorem 1.** The problem 1 for sk-algebra \( A = (K, X, Y, f) \) and elements \( x \in X \), \( y \in Y \) is polynomially equivalent to problem 1 for esk-algebra \( B = (K, W, g) \) and elements \( (x, t_1), (y, t_2) \in W \), \( t_1 = t_2 \), where \( W = (X \cup Y) \times \{0, 1\} \) and

\[
\begin{align*}
g_k(z, t) = \begin{cases} 
(f_k(z), t), & I_X(z) = 1, t_1 = 1, \\
(f_k^{-1}(z), t), & I_X(z) = 1, t_1 = 0, \\
(z, 1 - t), & \text{otherwise}.
\end{cases}
\end{align*}
\]

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**THE UPPER BOUNDS OF THE INTEGER DIFFERENTIALS AVERAGE PROBABILITIES FOR COMPOSITION OF THE KEY ADDER, SUBSTITUTION BLOCKS AND THE BLOCKSTRUCTURED LINEAR OPERATOR**

L. V. Kovalchuk², N. V. Kuchinska², V. T. Bezditnyi³

To estimate a block cipher resistance against differential cryptanalysis and its various modifications, as a rule, it is necessary to obtain the upper bounds of the round differential average probability. Round functions of most of the modern block encryption algorithms (e.g. AES, GOST 28147, “Kalina”) contain the composition of the key adder, substitution blocks, and the linear operator. Therefore, the problem of obtaining upper bounds for block cipher resistance is reduced to the problem of constructing upper bounds for the average probability of such compositions, or consists it as a subtask [1–3]. In this work, the upper bounds are obtained for the integer differentials average probability of maps which are compositions of the key adder, substitution blocks, and the blockstructured linear operator (over the same ring). The parameters of s-blocks, the bounds depend on, are defined and conditions, that ensure the least possible values of these parameters, are given. The results obtained, allow us to analyze the differential properties of the round function of block encryption algorithm and therefore the differential properties of the whole block encryption algorithm.

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ON STATISTICAL ESTIMATION OF MULTIVARIATE ENTROPY

U. Yu. Palukha¹, Yu. S. Kharin²

The problem of statistical estimation of Shannon entropy for \( n \)-words of random sequences is typical in cryptography, genetics and other applications [1, 2].

We have the sequence \( x_1, \ldots, x_T \in V = \{0, 1\} \), which is a stationary random sequence on the probability space \((\Omega, F, P)\). Let’s consider the circular sequence of length \( T + n - 1 \): \( x_{T+1} = x_1, \ldots, x_{T+n-1} = x_{n-1} \). We construct frequency estimator of \( p_j(n) = P(X^n_t = j^n_t) \), where \( j^n_t = (j_1, \ldots, j_n) \) is multiindex, and then build the entropy estimator by “plug-in” principle:

\[
\hat{h}(n) = - \sum_{j \in V_n} \hat{p}_j(n) \ln \hat{p}_j(n).
\]

Denote hypothesis \( H_0 = \{x_1\} \) is “true random”\( \Rightarrow \{p_j^1 = 2^{-n}, j_1^n \in V_n\} \).

**Theorem 1.** If \( \{x_t\} \) satisfies hypothesis \( H_0 \), then

\[
E_{H_0}[\hat{h}(n)] = h_0(T, n) = \sum_{m=1}^{n} e^{-\frac{T}{2^n}} \sum_{k=1}^{\infty} \frac{T^k \ln(k+1)}{2^{mk}k!}(e^{-\frac{T}{2^n}} - 2^k - 1).
\]

Theorem 1 is useful for construction of statistical tests to evaluate performance of the random number generators’ quality.

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CLASSICAL AND MODERN CRYPTOGRAPHY

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CUBE ATTACKS WITH LOW DEGREE MAXTERMS

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A new kind of algebraic attack on symmetric cryptosystems, called Cube attack, was proposed by Itai Dinur and Adi Shamir in [1,2]. It was originally designed to break stream cipher Trivium – a candidate on eStream competition held in 2008. But it was shown later that this kind of attack or its main ideas can be applied to break some block ciphers and message digest algorithms as well.

Cube attack complexity mainly depends on maxterm degree restrictions in a way that bigger maxterm degree leads to more expensive calculations. For real-world cipher output functions probability of having low degree maxterms is almost negligible, but an analysis of such functions is still an interesting problem related to Boolean functions theory.

In this paper, we present mathematical expression for number of Boolean functions, to which Cube attack with maxterms of degree 1 can be successfully applied. It depends on encryption function degree and on the number of its secret and public variables. Theoretical results were verified using computer calculations. We also provide analysis of cube attacks success rate for encryption functions with different numbers of variables.

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PROVABLE SECURITY OF SAFER-LIKE SUBSTITUTION-PERMUTATION NETWORKS AGAINST DIFFERENTIAL CRYPTANALYSIS

Serhiy Yakovliev

The SAFER family of block ciphers was proposed by James Massey as fast encryption algorithms suitable even for smart-cards. For now provable security against differential cryptanalysis for SAFER-like ciphers has been an open question.

Let $V_u$ be a linear space of $t$-bit vectors. Consider two operations $\circ$, $\bullet$, $i = 1, n$, so $(V_u, \circ)$ and $(V_u, \bullet)$ are abelian groups with zero vector as identity element. Introduced operations induce operations on $t$-bit $\circ$-differential probability of $r$-round SAFER-like SP-network $E$ ($r \geq 2$). Then following claims are true:

1) If $\circ = \oplus$, then $MDP(E) \leq \Delta^{B-1}$
2) If $L$ has a matrix form over $V_u$ and $B \geq m$, then $MDP(E) \leq \Delta^{B-1}$
3) In other cases $MDP(E) \leq (M - 1) \cdot \Delta^{B-1}$, where $M = \max\{\text{ord}_o(x), x \in V_u\}$

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RELIABILITY AND QUEUEING

JUSTIFICATION OF CONJECTURES ON EQUIDISTRIBUTION OF ARITHMETIC SEQUENCES

N. M. Glazunov

Let \( T_p(c, d) = \sum_{x=1}^{p-1} e^{2\pi i \left( \frac{cx + d}{p} \right)} \) be a Kloosterman sum. Here \( p \) is a prime, \( c, d \) are integers not congruent zero modulo \( p \). By A. Weil estimate \( T_p(c, d) = 2\sqrt{p}\cos \theta_p(c, d) \).

We will consider two possible distributions of angles \( \theta_p(c, d) \) on semiinterval \([0, \pi)\):

a) \( p \) is fixed and \( c \) and \( d \) vary over \( \mathbb{F}_p^* \); what is the distribution of angles \( \theta_p(c, d) \) as \( p \to \infty \);

b) \( c \) and \( d \) are fixed and \( p \) tends to \( \infty \) for all primes not dividing \( c \) and \( d \).

Theorem 1. For any sequence of prime finite fields \( \mathbb{F}_p \), when \( p \) tends to \( \infty \), \( 1 \leq c, d \leq p - 1 \); \( x, c, d \in \mathbb{F}_p^* \), the distribution of angles \( \theta_p(c, d) \) on semiinterval \([0, \pi)\) tends to the equidistribution with the density \( \frac{1}{2\pi}\sin^2 t \).

Similar theorems have been proved by N. Katz and by A. Adolphson for few other sums. Our proof is based on combining of P. Deligne [1] and N. Katz [2] methods. Next we proposed an approach to justification of conjecture b). This approach is based on methods of works [3, 4], Langlends program and our considerations [5].

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MIXED TYPE QUEUING SYSTEMS AS MATHEMATICAL MODELS OF RELIABILITY AND SURVIVABILITY

Revaz Kakubava¹, Archil Prangishvili², Grigol Sokhadze³

The given paper deals with the redundancy and maintenance problem for a wide class of any territorially distributed standby systems consisting of unreliable repairable components. Mathematical models for interaction of degradation and its compensation processes in these systems are proposed and their possible applications are partially analysed. These models represent mixed type queuing systems for two parallel maintenance operations – replacements and repairs. The problem for optimization of these system by economic criterion is stated. The possible ways of its solution are discussed.

The fact is that, during last decades, in reliability theory and practice (as well as in survivability theory and practice), the problems of redundancy, maintainability and supply of large scale territorially distributed systems, including terrestrial ones, are becoming the main directions.

At the same time, traditional maintenance models for these systems in many cases proved to be unsuitable, and there was an urgent need for the construction and investigation of entirely new types of models for the mathematical description of the mentioned technical systems.

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A GI/G/∞ QUEUEING PROCESS UNDER HIGH LOADING

I. N. Kovalenko

Let \( X(t) \) be a number of customers in the queue at time \( t \) and \( X_n \) be a number of customers just before the arrival of \( n \)th call. We investigate correlation properties of both processes in the high loading case. In particular, we study the difference between \( D/G/\infty \) and \( GI/G/\infty \) models.

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FAST SIMULATION OF STEADY-STATE PROBABILITIES OF $GI/G/\infty$ QUEUEING SYSTEM IN HEAVY TRAFFIC

Igor Kuznetsov

A queuing system $GI/G/\infty$ with recurrent input flow of customers determined by distribution function (d.f.) $F(x)$ with density function $f(x)$ being considered. There is an infinite number of servers with service time determined by d.f. $G(x)$. The system is investigated under heavy traffic condition when the mean time between successive customers is essentially less than the mean service time. We consider the presence of at least $r$ customers in the system as its failure. The main aim of the investigation is to evaluate the steady-state probabilities:

$$Q(r) = \lim_{t \to \infty} P\{\nu(t) \geq r\}, \quad Q^{(0)}(r) = \lim_{n \to \infty} P\{\nu(r(n) - 0) \geq r\}$$

where $\nu(t)$ is the number of customers at time $t$, and $\{r(n), n \geq 1\}$ is the sequence of successive moments of customer arrivals (it is obvious that $Q(r) = Q^{(0)}(r)$ in the case of Poisson input flow).

It is obvious that for the increasing values of $r$ the probabilities $Q(r)$ and $Q^{(0)}(r)$ vanish. Therefore, the usual Monte Carlo simulation is unable to produce the required accuracy of estimates for the large values of $r$. At the same time, the general form of d.f. $F(x)$ and $G(x)$ makes the use of analytical and even asymptotical approaches to determine the steady-state probabilities extremely difficult.

We suggest to evaluate the probabilities $Q(r)$ and $Q^{(0)}(r)$ using importance sampling that depends on some scale parameter. This approach makes it possible to produce asymptotically unbiased estimates. Numerical examples illustrate the variance reduction obtained by the method proposed.

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APPLICATION OF FAST SIMULATION IN SOLVING SOME COMPLEX PROBLEMS OF COMBINATORIAL ANALYSIS

Nickolay Kuznetsov

During the last three decades the fast simulation based on different variance reduction techniques proved to be a powerful tool in solving various difficult problems associated with the evaluation of small probabilities. Primarily it concerns the reliability theory and the queueing theory. At the same time similar approaches may be very efficient in solving difficult combinatorial problems of large dimension. It was quite unexpected that the fast simulation was able to give estimates of only 5% relative error while dealing with evaluation of a number of good permutations among the total number of $n!$ permutations for $n = 205$ [1]. The same high accuracy was obtained in estimating of a number of Latin squares for $n = 20 [2]$. These results may be significantly improved by applying modified fast simulation algorithms on SCIT-4 cluster of V. M. Glushkov Institute of Cybernetics. We propose algorithms for four combinatorial problems. The first one deals with the enumeration of so-called good permutations. A permutation $(s_0, \ldots, s_{n-1})$ is called good if the set $(t_0, \ldots, t_{n-1})$ formed according to the rule $t_i = i + s_i \pmod{n}$ is also a permutation. The second one deals with the evaluation of a number of Latin squares and rectangles for maximal values of $n$ and $m$ determining their dimension. And finally, the fast simulation algorithms to solve a knapsack problem and to evaluate the permanent of the matrix are proposed. The numerical examples illustrate the efficiency of the suggested algorithms.

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FAST SIMULATION OF THE RELIABILITY OF REPAIRABLE SYSTEM WITH TWO MODES OF OPERATION

N. Yu. Kuznetsov1, O. M. Khomyak2

Consider a system operating in two modes. System structures in these modes are described by different fault trees: Tree 1 and Tree 2. If we are talking about a nuclear reactor, the second mode can be interpreted as the reactor cooling mode. It starts when some undesirable event has occurred. The emergency mode begins, when TOP-event of Tree 1 (normal mode) occurs during the period of time $[0,T_1]$. The structure of system in the emergency mode is described

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A classical retrial queue with \( m \) servers is given by three parameters: \( \lambda \) is a rate of input Poisson flow of primary calls, \( \mu \) is a service rate and \( \nu \) is a rate of repeated attempts. For this model a service process \( Q(t) = (Q_1(t), Q_2(t)) \in S = \{0, 1, \ldots, m\} \times \{0, 1, \ldots\} \) is two-dimensional one where \( Q_1(t) \) is a number of occupied servers at \( t \geq 0 \) moment of time, \( Q_2(t) \) is a number of retrial calls.

In this presentation it is assumed that the parameters \( \lambda, \mu, \nu \) may be depended on the moving phase state \((i, j) \in S\) of the service process \( Q(t) \): \( \lambda = \lambda_{ij}, \mu = \mu_{ij}, \nu = \nu_{ij} \). From a mathematical standpoint the assumed dependence complicates the retrial queue model. In this case the method of generating functions which proved its effectiveness for the classical models (see, for example, [1]) is not applicable.

In the proposed work for models of retrial queues with variable local parameters the following aspects are considered:
1) the existence conditions of stationary regime; 2) possibility of construction of explicit closed formulas for stationary probabilities through model parameters; 3) development of effective recurrent schemes for characteristics of multi-channel queues; 4) optimal control making for local parameters in a set of threshold and hysteresis strategies.

A class of models under consideration extends the field of applications. Research methods proposed in the work make possible to set and solve problems which are related to income maximization for queuing systems introduced above ([2,3]).

The group of \( N \) identical repairable systems is considered. Failed elements of each system are to be repaired. We assume that the stream of repairing demands from every system is Poisson with parameter \( \lambda N^{-1} \). Every failed element immediately enters the unlimited repair facility so that its repairing starts immediately. Each element after it has been repaired returns into that system which has the largest group of faulty elements. Let \( G(x) = P\{\eta \geq x\} \) be the distribution function of random repairing time of element. Repairing times of different elements are independent and identically distributed. We use the notation \( m_1 = \int x dG(x) \). Let \( \rho = \lambda m_1 \) be the total load to repair facility formed by all systems. Let \( \Lambda_j(r_0) \) be the refuse intensity of the system \( j \), and \( \Lambda(r_0) \) be the total intensity of all group.

**Theorem 1.** If \( m_1 < \infty \) and \( n \geq 2 \). Then for \( \frac{\rho}{N} \to 0 \) we have

\[
\Lambda_j(r_0) < C \sim \frac{\lambda}{N} \left( \frac{\rho}{N} \right)^n \exp(-\rho) \sum_{k \geq 0} \frac{\rho^k}{k!(k+2)\ldots(k+n)},
\]

\[
\Lambda(r_0) < NC \sim \lambda \left( \frac{\rho}{N} \right)^n \exp(-\rho) \sum_{k \geq 0} \frac{\rho^k}{k!(k+2)\ldots(k+n)},
\]
For example, consider $N = 10^9$, $\rho = 10^6$ and $n = 10$. Let every element after repairing returns into that system which has the largest set of fault elements instead of its returning into its initial system. Then the intensivity of systems and groups failure becomes more then $2.6 \times 10^{47}$ times less!

APPLICATIONS OF SPACE MERGING ALGORITHMS IN TELETRAFFIC THEORY

Agassi Melikov$^{1}$, Leonid Ponomarenko$^{2}$

The most radical approach to overcoming complexity of the analysis of real complex telecommunication systems consists in construction of more simple merged system which analysis essentially is easier than the analysis real, and the basic characteristics can be accepted as characteristics of last [1].

In the present work the methods and algorithms to merging of state space of complex teletraffic systems are uniformly developed. The idea of these methods consists that the state space of real system is split on finite or infinite number of disjoint classes. States of each of these classes are merged in one state. In the new merged state space the merged system which functioning in the certain sense describes functioning initial system is under construction.

The developed numerical methods can be applied to research both classical models of wired and wireless networks (CWN) and multi-rate models of multimedia communication networks with arbitrary number of heterogeneous calls. Thus efficiency of the offered methods is shown on concrete classes of teletraffic systems [2,3].

References


ON THE ANALYSIS OF THE QUEUE LENGTH IN OPEN QUEUEING NETWORKS

S. Minkevičius$^{1}$, E. Greičius$^{2}$

Modern queueing theory is one of the tools for a quantitative and qualitative analysis of communication systems, computer networks, transportation systems, and many other technical systems. This paper is designated for the analysis of queueing systems, arising in the network and communications theory (called an open queueing network). Heavy traffic limit theorems on the queue length of customers in an open queueing network are presented in this paper and an application of the proved theorem on the reliability of computer networks is provided at the end of the paper.

One of the main trends of research in queueing theory is related with the asymptotic analysis of explicit formulas or equations that describe the distribution of characteristics of a queueing system. To make an analysis of this kind, we certainly assume the existence itself of such explicit formulas or equations, and, in addition, an unrestricted approximation of a queueing system to some limit.

The first results on the behavior of single-server queueing systems in heavy traffic were obtained by J. Kingman in 1962, (see [2,3]). Later on, there appeared many works designated to the various aspects of diffusion approximations of the models of a queueing system. The main tool for the analysis of queueing systems in heavy traffic is a functional probability limit theorem for a renewal process (the proof can be found in [1]).

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ON MULTICRITERIA OPTIMIZATION OF FINITESOURCE RETRIAL QUEUES

V. D. Ponomarov

This presentation deals with the problem of optimization of total income in the finitesource queueing system with repeated calls and the service rate that depends on the current system state. At any moment of time the service rate is fully defined by a hysteresis control policy. We will define a multicriteria optimization problem in terms of stationary probabilities of the system, find explicit representations of this probabilities through the system parameters and describe the algorithm, which gives an optimal solution in the chosen class of control policies.

The system state at any time \( t \) can be described by means of a trivariate process

\[
X(H_1, H_2, t) = \{(C(H_1, H_2, t); N(H_1, H_2, t); R(H_1, H_2, t)) \mid t \geq 0 \},
\]

where \( C \) is the number of busy servers, \( N \) is the number of retrials and \( R \) is the system operating mode. The process \( X(H_1, H_2, t) \) is a regular continuous time Markov chain with \( S(X) = \{0, \ldots, c\} \times \{0, \ldots, m - c\} \times \{1, 2\} \) as the state space.

Solution of the following multicriteria problem gives a control policy that maximizes the total income from the given system

\[
f_i(H_1, H_2) \to \max, i = 1, 2, \\
f_i(H_1, H_2) \to \min, i = 3, 4,
\]

where \( f_i(H_1, H_2) \) is the number of calls that were served by the system while working in the \( i \)-th mode, \( f_3(H_1, H_2) \) is the number of calls that were rejected by the system and became the sources of retrial calls, \( f_4(H_1, H_2) \) is the number of service rate switches.

Functionals \( f_i(H_1, H_2), i = 1, \ldots, 4 \) can be expressed in terms of stationary probabilities \( \pi_i(r) \) of the \( X(H_1, H_2, t) \). This probabilities can be found in the explicit vector-matrix form through the system’s parameters, which gives an explicit solution of the formulated optimization problem.

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LOWER BOUNDS FOR THE PROBABILITY OF A SYSTEM FAILURE ON THE TIME INTERVAL UNDER INCOMPLETE INFORMATION ABOUT DISTRIBUTION FUNCTION OF THE TIME TO FAILURE

Lidiya S. Stojkova

The upper and lower bounds for the functional \( I(F) = \int_{u}^{v} dF(x), \ 0 < u < v < \infty \), were obtained by A. A. Markov (1895) without the assumption that the distribution function \( F(x) \) has unimodal density. The earliest results with unimodal density in a special case were obtained by Gauss (1821). Various problems concerning inequalities with unimodal distribution function were considered in the articles of S. Verblunsky (1936), N. L. Jonson and C. A. Rogers (1951), H. L. Royden (1953), B. Ulin (1953), C. L. Mallows (1956) et al. (see references [1–3]).

We consider the problem of finding the greatest lower bounds for the functional \( I(F) \) on the set of distribution functions \( F(x) \) of nonnegative random variables with the first and the second given moments and the unimodal density with the mode \( m > 0 \) for the case \( m < u \).

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A PROBABILITY MODEL OF THE WORK OF AN ATM

V. N. Turchyn¹, I. S. Bondarenko²

The problem of simulation of the automated teller machine (ATM) work is considered in order to answer the question: “How much cash should be loaded into an ATM?”

A model of ATM work is constructed as a queueing system which minimizes the costs of ATM service and provides a predetermined quality of customer service: probability of a failure to serve a client does not exceed a given number.

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Consider a single-server queueing system with an infinite queue. The state of the system at time $t$ is a vector $(n_t, x_t, y_t) \overset{def}{=} X_t$, where $n_t$ is the number of customers in the system and $x_t$ is the elapsed time of the current service, $y_t$ is the elapsed time of the current waiting for a new customer; if $n_t = 0$, then $x_t = 0$. Both service time and incoming flow are described by two intensities $h$ and $\lambda$ respectively, which depend on the state of the system $X_t$: nonrigorously, this is understood so as on any nonrandom interval of time $[t, t + \Delta)$ the service of current serving will be completed with probability $h(X_t)\Delta + o(\Delta)$ and a new customer will arrive with probability $\lambda(X_t)\Delta + o(\Delta)$; group arrivals are not allowed. The Markov process $X_t$ is defined on the state space $\mathcal{X} = \mathbb{Z}_+ \times \mathbb{R}_+^2$. Denote by $\mu_t^{X_0}$ the distribution of $X_t$ at time $t$. The following Theorem is based on and extends results from [1].

**Theorem 1.** Let the functions $\lambda$ and $h$ be Borel measurable and bounded; for any $X = (n, x, y) \in \mathcal{X}$, $h(X) \geq \frac{\kappa}{1 + x}$, $\lambda(X) \leq \Lambda < \infty$, $\inf_y \lambda(0,0,y) \geq \lambda_0 > 0$, and $\kappa > 4(1 + 2\Lambda)$, and let the value $k > 1$ be such that $k > 2^{k+1}(1 + \Lambda 2^k)$. Then there exists a unique stationary measure $\mu$ and computable constants $m > k$ and $C_1, C_2 > 0$ such that $\|\mu_t^{X_0} - \mu\|_{TV} \leq \frac{C_1(1 + n_0 + x_0)^m + C_2}{(1 + t)^{k+1}}$ for any $t \geq 0$.

**References**


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ON SOME ESTIMATES OF MEAN RESIDUAL LIFE FUNCTION UNDER RANDOM CENSORING FROM THE RIGHT

A. A. Abdushukurov¹, R. S. Muradov², K. S. Sagidullayev³

In survival analysis our interest focuses on a nonnegative random variables (r.v.’s) denoting death times of biological organisms or failure times of mechanical systems. A difficulty in the analysis of survival data is the possibility that the survival times can be subjected to random censoring by other nonnegative r.v.'s and therefore we observe incomplete data. There are various types of censoring mechanisms. The estimation of distribution function of lifetime and its functionals from incomplete data is one of the main goals of statisticians in survival analysis. In this article we consider only right censoring model and problem of estimation of mean residual life function both in independent and dependent censoring cases(for details, refer to [6]) assuming that the dependence structure is described by known copula function(see [7]). For survival function we use relative-risk power estimator(see [3]) and its extension to dependent censoring case.

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FORECASTING OF REGRESSION TIME SERIES UNDER CLASSIFICATION OF THE DEPENDENT VARIABLE

H. Ageeva¹, Yu. Kharin²

Different distortions, such as censoring, rounding, grouping [1], are frequently observed in real data [2]. Consider a special case of grouped data. Let

\[ Y_t = F^0(X_t, \theta^0) + u_t, \quad t = 1, \ldots, n, \]

be a regression model, where \( X_t \in X \subseteq \mathbb{R}^n \), \( t = 1, \ldots, n \), are independent regressors, \( F^0(\cdot) : X \times \Theta \rightarrow \mathbb{R}^1 \) is some regression function specified by a parameter \( \theta^0 \in \Theta \subseteq \mathbb{R}^m \), \( \{u_t\}_{t=1}^n \) are i.i.d. Gaussian random variables, \( \mathcal{L}(u_t) = \mathcal{N}(0, (\sigma^0)^2) \). We a given a set of \( K \) nonintersecting intervals:

\[ A_1 = (a_0, a_1), \quad A_2 = (a_1, a_2), \ldots, A_{K-1} = (a_{K-2}, a_{K-1}), \quad A_K = (a_{K-1}, a_K), \quad -\infty = a_0 < a_1 < \cdots < a_K = +\infty. \]

We do not observe true values of \( \{Y_t\}_{t=1}^n \). Instead we observe new values \( \{\nu_t\}_{t=1}^n \), where \( \nu_t = k \), if \( Y_t \in A_k \), \( k \in \{1, 2, \ldots, K\} \).

Under this grouping distortion of the dependent variables we present the following results:

- identifiability conditions for the parameters \( \theta^0, (\sigma^0)^2 \);
- plug-in forecasts and their properties;
- statistical tests on the belonging of \( F^0(\cdot) \) to some parametric family;
- results of computer experiments.

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CONFIDENCE INTERVALS FOR UNKNOWN RESPONSE FUNCTIONS OF LINEAR SYSTEMS

I. P. Blazhievskaya

Let \( H = (H(\tau), \tau \in \mathbb{R}) \) be a response function of a time-invariant linear system. The inputs \( X_{\Delta} = (X_{\Delta}(t), t \in \mathbb{R}), \Delta > 0 \), are supposed to be real-valued stationary centered a.s. continuous Gaussian processes, that “close” to a white noise \((\Delta \to \infty)\). A response of this system on \( X_{\Delta} \) is described by the following a.s. continuous process

\[
Y_{\Delta}(t) = \int_{-\infty}^{\infty} H(t-s)X_{\Delta}(s)ds, \quad t \in \mathbb{R}.
\]

We consider the integral-type sample input-output cross-correlograms

\[
\hat{H}_{T,\Delta}(\tau) = \frac{1}{cT} \int_{0}^{T} Y_{\Delta}(t+\tau)X_{\Delta}(t)dt, \quad \tau \in \mathbb{R},
\]

as estimators for the unknown real-valued function \( H \in L_2(\mathbb{R}) \) \((c\) is some constant\). Along with \( \hat{H}_{T,\Delta} \), we investigate as \( T, \Delta \to \infty \) the asymptotic behavior of the error term

\[
\bar{W}_{T,\Delta}(\tau) = \sqrt{T}[\hat{H}_{T,\Delta}(\tau) - H(\tau)], \quad \tau \in \mathbb{R}.
\]

Using the asymptotic normality of \( \hat{H}_{T,\Delta} \) and \( \bar{W}_{T,\Delta} \) in spaces of continuous functions (see, [1]), we construct the confidence bands for the limiting process. To obtain main results, we apply the properties of quadratically Gaussian processes and classical theorems about the estimators of the tails of Gaussian distribution [2].

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HYPOTHESES ON FUNCTIONAL MOMENTS IN MIXTURE MODEL WITH VARYING CONCENTRATIONS

O. V. Doronin

We consider the series of samples \( \xi_{j,N}, \ j = 1, N, N \geq 1 \) from some measurable space \( \mathcal{X} \) with \( \sigma \)-algebra \( \mathcal{G} \). The distribution of each observation is given by formula \( \mathcal{P}[\xi_{j,N} \in A] = \sum_{m=1}^{M} p_{j,N}^{m} F_{m}(A) \) where \( A \in \mathcal{G}, p_{j,N}^{m} \) is the known concentration of the \( m \)-th component for observation \( \xi_{j,N} \), and \( F_{m} \) is the unknown CDF of the \( m \)-th component.

Let \( g_{k} \) be some measurable functions from \( \mathcal{X} \) to \( \mathbb{R}^{d_{k}}, k = 1, K, K \leq M \). Denote by \( \tilde{g}_{k}^{m} \) the functional moment of the \( m \)-th component: \( \tilde{g}_{k}^{m} := \int g_{k}(x)F_{m}(dx) \). We develop method for testing hypotheses of the form \( \mathcal{T}(\tilde{g}_{1}^{1}, ..., \tilde{g}_{K}^{K}) = 0 \) where \( T \) is some measurable function from \( \mathbb{R}^{d_{1} \times ... \times \mathbb{R}^{d_{K}}} \) to \( \mathbb{R}^{K} \). This method is based on the asymptotic properties of appropriate test statistic (consistency and asymptotic normality). Derived approach is tested on simulated samples.

Proposed method can be used for the analysis of medical, biological, sociological, politological, economical data.

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LA(M)N PROPERTY OF LÉVY PROCESS OBSERVED AT HIGH FREQUENCY

D. O. Ivanenko

We consider the Lévy process \( X^{\theta} \) described by

\[
X^{\theta}_{t} = \beta t + \gamma Z_{t} + U_{t}, \quad t \geq 0,
\]

where \( Z \) is a locally \( \alpha \)-stable symmetric Lévy process and \( U \) is a Lévy process which is independent of \( Z \), less active than \( Z \), and regarded as a nuisance process.
We assume that the process $X^j$ is observed at the points \{t_{k,n} = kh_n, k = 1, \ldots, n\}, i.e. at the uniform partition of the time interval [0, T_n], T_n := nh_n. We concentrate on high-frequency case, i.e. $h_n \to 0$. Our goal is to study the LAN (LAMN) property about the explicit parameter $\theta = (\gamma, \beta)$. Technically, this is if the likelihood ratio processes admit a certain quadratic expansion.

The general sufficient condition of LAN is obtained for a statistical model based on a discrete observations of a Markov process. We combine this result with the Malliavin calculus-based integral representations for derivatives of log-likelihood function to prove the LAN property for the model above.

The talk partially contains results obtained in a collaboration with A. Kulik and H. Masuda.

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ON THE EXTREME RESIDUAL IN REGRESSION MODELS

O. V. Ivanov\textsuperscript{1}, I. K. Matsak\textsuperscript{2}, S. V. Polotskiy\textsuperscript{3}

We consider linear regression

$$y_j = \sum_{i=1}^{q} \theta_i x_{ji} + \epsilon_j, \quad j = 1, n,$$

where $\epsilon_j$ are i.i.d.r.v., $\mathbb{E}_{\epsilon_j} = 0$. Let $\hat{\theta}$ be an estimator of unknown parameter $\theta$,

$$\hat{y}_j = \sum_{i=1}^{q} \hat{\theta}_i x_{ji}, \quad \hat{\epsilon}_j = y_j - \hat{y}_j, \quad \hat{Z}_n = \max_{1 \leq j \leq n} \hat{\epsilon}_j, \quad \hat{Z}_n^* = \max_{1 \leq j \leq n} |\hat{\epsilon}_j|.$$

Classical theorems of regression analysis describe various properties of residual sum of squares. Instead, in the present lecture we study asymptotic behavior of the r.v. $\hat{Z}_n, \hat{Z}_n^*$. We choose the least squares and minimax estimators as the estimators of $\theta$.

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ASYMPTOTIC PROPERTIES OF M-ESTIMATORS IN NONLINEAR REGRESSION WITH DISCRETE TIME AND SINGULAR SPECTRUM

Alexander V. Ivanov\textsuperscript{1}, Igor V. Orlovsky\textsuperscript{2}

Consider a regression model

$$X_j = g(j, \theta) + \epsilon_j, \quad j = 1, N,$$

where $g(j, \theta)$ is a nonrandom parametric sequence on $\Theta_\gamma$, $\Theta_\gamma = \bigcup_{|a| \leq 1} (\Theta + a\gamma)$, $\gamma > 0$ is some number, $\Theta \subset \mathbb{R}^q$ is bounded open convex set, and $\epsilon_j = G(\xi_j), \quad j \in \mathbb{Z}, \quad G(x), \quad x \in \mathbb{R}^1$, is some Borel function, $E\epsilon_0 = 0, E\epsilon_0^2 < \infty; \xi_j, \quad j \in \mathbb{Z},$
is stationary Gaussian sequence with $E\xi_j = 0$ and covariance function $B(j) = E\xi_0\xi_j = \sum_{i=0}^r A_i \cos(\chi_j), r \geq 0$, where $0 \leq \chi_0 < \chi_1 < \ldots < \chi_r$, $0 < \alpha_i < 1$, $i = 0, r$, $\sum_{i=0}^r A_i = 1$, $A_i \geq 0$.

**Definition 1.** $M$-estimator of unknown parameter $\theta \in \Theta$, with loss function $\rho(x)$, $x \in \mathbb{R}^1$, obtained by the observations $X_i$, $j = 1, N$, is said to be any random vector $\hat{\theta}_N \in \Theta^\circ$, having the property $S(\hat{\theta}_N) = \inf_{\tau \in \Theta} S(\tau)$, $S(\tau) = \sum_{j=1}^N \rho(X_j - g(j, \tau))$.

Consistency, asymptotic uniqueness and asymptotic normality of $M$-estimator are considered in the talk. Key moment of asymptotic normality proof is an application of central limit theorem from [1].

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**STATISTICAL ANALYSIS OF CONDITIONAL AUTOREGRESSIVE MODELS BASED ON SPATIO-TEMPORAL DISCRETE DATA**

Yu. S. Kharin¹, M. K. Zhurak²

Models based on spatio-temporal data become widely used for solving practical problems in meteorology, economics, medicine and other fields [1]. To describe the discrete data models based on Poisson probability distribution are often used. Unfortunately, in most cases the spatial dependence is not taken into account in the existing Poisson models of discrete data. To fill this gap we develop the new Poisson conditional autoregressive model for spatio-temporal data. In the case of finite space of states we also develop the new Binomial conditional autoregressive model. These models can be used to describe the discrete spatio-temporal data, that arises in space-time monitoring of the incidence rate in a specific geographical area.

Studying the probabilistic properties we prove that under the Poisson and Binomial conditional autoregressive models the observed process is the nonhomogeneous vector Markov chain with countable and finite space of states respectively. The formulas for calculation of the one-step transition probability matrix, expectation and variance are built. In case of unknown parameters the “plug-in” principle is applied. Risk values for constructed forecasting statistics are calculated. The computer experiments are carried out on simulated and also on real medical data that describes the spatio-temporal incidence rate of children leukemia.

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**SMOOTHING TIME SERIES BY A LINEAR SPLINE WITH THE UNKNOWN POINTS OF SWITCHING**

A. Korkhin

Let us consider time series $y_t = s_t + \varepsilon_t$, $t = 1, \ldots, T$, where $s_t$ is a linear spline function; $\varepsilon_t$, $t = 1, \ldots, T$ is sequence of the independent centered, normally distributed random values, $T$ is a length of a observation interval. It is assumed that the points of switching and their number are unknown.

We will designate the unknown vectors of spline coefficients and points of switching accordingly

$$\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{n+1}^0]$$ $\in \mathbb{R}^{2(n+1)}$, $\theta = [\theta_0, \theta_1, \ldots, \theta_n]^T$.

Here the character "T" means transposition; $\alpha_0, \alpha_1, i = 1, \ldots, n + 1$ are unknown coefficients of $i$-th segment.

We will designate $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{n+1}]^T \in \mathbb{R}^{2(n+1)}$ and $\theta = [\theta_0, \theta_1, \ldots, \theta_n]^T$ are varied vectors of spline coefficients and points of switching accordingly. Put $U_t(t_{i-1}, t_i) = 1$ if $t_{i-1} < t \leq t_i$, $U_t(t_{i-1}, t_i) = 0$ otherwise, $i = 1, \ldots, n + 1$, where $t_0 = 0, t_{n+1} = T$, and $s_t = \alpha_0 + \alpha_1 t$, $t = 1, \ldots, T, i = 1, \ldots, n + 1$.

Then we obtain function regression $s(t, \alpha, \theta) = \sum_{i=1}^{n+1} s_t U(t_{i-1}, t_i)$. Its parameters are mixed: $\alpha$ is continuous, $\theta$ is discrete. We will estimate vectors $\alpha$ and $\theta$ on steps.

**Step 1.** Let $n = [T/3]$, where $[a]$ is whole part of number $a$. $3$ is minimum superfluity for the evaluation of segments. We will find estimations $\alpha$ and $\theta$ as solution of the problem

$$\sum_{t=1}^{T} (y_t - s(t, \alpha, \theta))^2 \Rightarrow \min$$
under parameter restrictions
\[(a_0 - a_{0,i+1}) + (a_{1i} - a_{1,i+1})t_i = 0, \quad t_i \geq t_{i-1} + 3, \quad i = 1, \ldots, n.\]

Here minimization is executed on \(\alpha\) and \(\theta\).

The formulated problem of nonlinear evaluation with restrictions is problem of minimization on the mixed variables. Its a objective function is undifferentiated. In paper properties of this problem are considered. The problem can be solved by the methods, developed Lasdon, Sergyenko, Shylo. The numeral solution can be obtained with the use of processors MS Excel, Mathcad.

Step 2. Verification of statistical hypotheses about equality of slopes two or more segments of spline. Receipt of the revised value \(n\) with the required probability. Solution of the evaluation problem for the revised \(n\).

The example of solution of the practical task is described. The described method can be extended on other types of splines.

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DISCRETE MODEL OF MARKOV DIFFUSION WITH STATISTICALLY PERSISTENT LINEAR REGRESSION

D. Koroliouk

Discrete Markov diffusion \(\alpha_t, t \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}\), is determined by a solution of difference stochastic equation for increments \(\Delta\alpha_t := \alpha_{t+1} - \alpha_t:\n\Delta\alpha_t = -V\alpha_t + \sigma\Delta\alpha_t, \quad t \in \mathbb{N}_0, \quad E\Delta\alpha_t = 0, \quad E[\Delta\alpha_t]^2 = 1, \quad \forall t \geq 1.\n
and is characterized by its initial value \(\alpha_0\) and by two real parameters:
\[V: \text{the regression of the predictable component, } 0 < V < 2;\n\sigma^2: \text{the variance of stochastic component } \Delta\alpha_t, t \geq 1, \text{ defined by a sequence of independent, identically distributed Gaussian random variables. The initial value } \alpha_0 \text{ is also supposed to be independent respect to } \Delta\alpha_t, t \geq 1.\n
The joint normal distribution of two-component vector \((\alpha_t, \Delta\alpha_t), t \geq 0\), is represented by the covariance matrix (Theorem 1) under natural conditions of gaussianity and stationarity of the discrete Markov diffusion.

The covariances of discrete Markov diffusion generate consistent (in mean square) statistics for two real-valued parameters \(V\) and \(\sigma^2\) (Theorem 2).

The same problems are considered for multi-dimensional discrete Markov diffusion.

The stationary sequence simulation of discrete Markov diffusion is reproduced for two values set of parameters \(V\) and \(\sigma^2\).

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ESTIMATION OF PARAMETERS OF SDE DRIVEN BY FRACTIONAL BROWNIAN MOTION

K. Kubilius

Recently, much attention has been paid to SDEs driven by the fractional Brownian motion (fBm) and to the problems of the statistical estimation of model parameters. There are a few results concerning the estimation of the Hurst index if the observed process is described by a SDE driven by fBm. In general, only a strong consistency of these estimators was proved. We concentrate on the estimates of the Hurst parameter and volatility of the nonlinear reducible SDE driven by fBm. The obtained estimators are strongly consistent and asymptotically normal. Thus, we can construct confidence intervals for the Hurst parameter and volatility. Verhurst, Gompertz, and Landau-Ginzburg equations are examples of reducible SDEs. The estimators of parameters were constructed from discrete observations of the trajectory of the solution of the SDE by using a quadratic variation of the second order increments.

If observed process is Ornstein-Uhlenbeck then the exact confidence intervals for the Hurst parameter based on a single observation of a discretized sample path are obtained.

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SIMULTANEOUS ESTIMATORS IN COX PROPORTIONAL HAZARDS MODEL WITH MEASUREMENT ERROR

A. Kukush

Cox proportional hazards model for lifetime $T$ is considered. Simultaneous estimators $\lambda_n(\cdot)$ and $\beta_n$ of baseline hazard rate $\lambda(\cdot)$ and regression parameter $\beta$ are studied. $\lambda(\cdot) \in \Theta_\lambda \subset C[0,\tau]$, $\tau > 0$, where $\Theta_\lambda$ is a compact convex subset of positive Lipschitz continuous functions. An i.i.d. sample of triples $(Y_i, \Delta_i, W_i)$, $i = 1, \ldots, n$, is observed, where $Y_i$ is right-censored value of lifetime $T_i$ with i.i.d. censors $C_i$'s distributed in $[0,\tau]$ with unknown distribution, and $\Delta_i = I(T_i \leq \tau)$ is a censorship indicator, $\tau$ is right endpoint of censor distribution, and $W_i$'s are values of vector regressors $X_i$'s contaminated by i.i.d. errors $U_i$, $W_i = X_i + U_i$, with known moment generating function $M_U(\beta) := E e^{\beta^T U}$. The estimators maximize the corrected log-likelihood function

$$Q_n^{cor}(\lambda, \beta) := \frac{1}{n} \sum_{i=1}^{n} \left( \Delta_i \left( \log \lambda(Y_i) + \beta^T W_i \right) - \frac{e^{\beta^T W_i}}{M_U(\beta)} \int_{0}^{\tau} \lambda(u) du \right).$$

The estimators are strongly consistent. Under reasonable assumptions,

$$\sqrt{n}(\beta_n - \beta_0) \xrightarrow{d} N_k(0, \Sigma_\beta), \int_{0}^{\tau} (\lambda_n - \lambda_0)(u)f(u)G_C(u) \xrightarrow{d} N_j(0, \sigma_j^2),$$

where $f$ is an arbitrary Lipschitz continuous function. The $\Sigma_\beta$ and $\sigma_j^2$ are evaluated explicitly. The corresponding asymptotic confidence regions can be constructed using the Kaplan-Meier estimator of the censor survivor function $G_C$. A way to compute the estimators is discussed based on the fact that $\lambda_n(\cdot)$ is a linear spline. The results are joint with MS student C. Chimisov (Kyiv) and published in [1].

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NONPARAMETRIC STATISTICS OF FINITE MIXTURES

R. E. Maiboroda

In finite mixture models (FMM) it is assumed that each observed subject belongs to one of $M$ subpopulations (mixture components) and the distribution of the observed variable $\xi$ of a subject depends on the subpopulation to which it belongs. So the unconditional distribution of the $j$-th observation is

$$P\{\xi_j \in A\} = \sum_{i=1}^{M} p^i F_i(A), \quad j = 1, \ldots, n,$$

where $F_i$ is the distribution of $\xi$ at the $i$-th component, $p^i$ is the probability that the observed subject belongs to the $i$-th component (mixing probability or concentration). In the case when no additional assumptions are made on $F_i$, the problem of $F_i$ estimation is unidentifiable in the model (1) even if $p^i$ are known.

In the communication we will consider some modifications of FMM which allow consistent estimation of $F_i$. The first way to obtain consistent estimates is to make additional assumptions on the shape of $F_i$, such as symmetry or coordinatewise independence.

Another way to make the model (1) identifiable is to assume that the mixing probabilities vary from observation to observation. This leads to the model of mixture with varying concentrations (MVC)

$$P\{\xi_j \in A\} = \sum_{i=1}^{M} p^i_j F_i(A).$$

We will consider estimation and hypotheses testing in MVC and FMM with symmetric components. Examples of real life data analysis will be presented.

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We consider the Samuelson model with a telegraph drift [2] which is a modification of the Samuelson model for a financial asset price dynamics [1]. In the modified model the diffusion coefficient \( \mu \) is constant and the drift coefficient \( \mu(t) \) is an integral of a generalized telegraph signal: \( \mu(t) = \int_0^t \eta(s)ds \), where \( \{\eta_k\}_{k=0}^{\infty} \) is a sequence of independent random variables, normally distributed with parameters \( (0; \theta^2) \); \( \nu(t) \) is an independent Poisson process with a parameter \( \lambda > 0 \). We are interested in estimating the unknown parameters \( \lambda, \theta, \sigma \) for the defined model. Denoting by \( S(t) \) the financial asset price at time \( t \), we observe values \( S_k = S(kh) \) at equidistant times \( t_k = kh \), \( k = 0, 1, \ldots, n \) and set \( z_k = \ln \frac{S_{k+1}}{S_k} \). The sequence \( \{z_k\}_{k=0}^{\infty} \) was proved to be strictly stationary and ergodic [2]. This enabled us to obtain strongly consistent estimates and asymptotic confidence regions for the parameters \( \lambda, \theta, \sigma \).

We deal with the problem of optimal estimation of the functional \( A_N \xi = \sum_{i=0}^{N} a_i \xi_i \), that depends on the unknown values of a harmonizable symmetric \( \alpha \)-stable random sequence \( \{\xi_n, n \in \mathbb{Z}\} \) from observations of the sequence \( \{\xi_k + \eta_k, k \in \mathbb{Z}\} \) at points of time \( k \in \mathbb{Z} \setminus \{0, 1, \ldots, N\} \). Harmonizable symmetric \( \alpha \)-stable random sequences \( \{\xi_n, n \in \mathbb{Z}\} \) and \( \{\eta_n, n \in \mathbb{Z}\} \) are supposed to be mutually independent and have spectral densities \( f(\theta) > 0 \) and \( g(\theta) > 0 \) satisfying the minimality condition. It is shown that spectral characteristic \( h(\theta) \) of the optimal estimate is determined by equation

\[
\left( \sum_{j=0}^{N} a_j e^{ij\theta} - h(\theta) \right)^{<\alpha-1>} f(\theta) - (h(\theta))^{<\alpha-1>} g(\theta) = \sum_{j=0}^{N} c_j e^{ij\theta},
\]

where \( c_j \) are unknown coefficients determined from the system of equations

\[
\int_{-\pi}^{\pi} e^{-ijk} (h(\theta))^{<\alpha-1>} (f(\theta) + g(\theta))d\theta = 0, \quad k = 0, 1, \ldots, N.
\]

In the case where spectral densities \( f(\theta) \) and \( g(\theta) \) are not known, but a set of admissible spectral densities is given, we propose relations that determine least favorable spectral densities and minimax spectral characteristic for optimal estimates of the functional \( A_N \xi \). For more details on minimax estimation problems see [1–3].

**REFERENCES**

As estimator of $B$ we choose correlogram built by the residuals $\hat{X}(t) = X(t) - g(t, \hat{\theta}_T)$, $t \in [0, T + H]$, namely:

$$B_T(z, \hat{\theta}_T) = T^{-1} \int_0^T \hat{X}(t + z)\hat{X}(t)dt, \quad z \in [0, H],$$

where $H > 0$ is a fixed number, $\hat{\theta}_T$ is the least squares estimator of unknown parameter $\theta$.

Under some assumptions on regression function $g(t, \theta)$ and spectral density $f(\lambda)$ of process $\varepsilon(t)$ we prove that

$$X_T(\cdot) = T^{1/2} \left( B_T(\cdot, \hat{\theta}_T) - B(\cdot) \right) \xrightarrow{D} Y$$

in the space of continuous functions $C[0, H]$, where $Y$ is a Gaussian process with zero mean and covariance function $b(h_1, h_2) = 4\pi \int_0^\infty f^2(\lambda) \cos \lambda h_1 \cos \lambda h_2 d\lambda, \ h_1, h_2 \geq 0$.

The proof of this fact is based on the methodological machinery of the books [1, 2].

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ON INTEGRAL FUNCTIONALS OF A DENSITY

E. A. Nadaraya$^1$, G. A. Sokhadze$^2$

Consider the functional of following type

$$I(f) = \int_{-\infty}^{\infty} \varphi(x, f(x), f'(x), ..., f^{(m)}(x))dx,$$

where $\varphi$ is a smooth function of $m + 2$ variables; $f(x)$ is an unknown probability distribution density of a random variable $X$; $f^{(k)}(x)$, $k = 0, 1, ..., m$ is a derivative of the function $f(x)$ of the order $k$. Let $X_1, X_2, ..., X_n$ be a sample of independent identically distributed random variables each of which has a distribution coinciding with the distribution of $X$.

The problem of statistical estimation of the functional $I(f)$ will be studied on the basis of this sample using the truncated plug-in-estimator:

$$I(\hat{f}_n, s_n) = \int_{-s_n}^{s_n} \varphi(x, \hat{f}_n(x), \hat{f}'_n(x), ..., \hat{f}^{(m)}_n(x))dx,$$

where $\hat{f}_n(x)$ is the estimator of the density $f(x)$, and $f_n^{(k)}(x), k = 0, 1, ..., m$, is the derivative of the function $\hat{f}_n(x)$ order $k$, $\hat{f}_n^{(0)}(x) \overset{def}{=} \hat{f}_n(x)$. Theorems of consistency and asymptotic normality are proved. The results extended theorems from [1–4].

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GUARANTEED LINEAR MEAN SQUARED ESTIMATES OF RANDOM FIELDS

A. G. Nakonechnyi

Let realization of the random field $\Theta(x)$ be observed in measurable by Lebesgue set $D$ of space $\mathbb{R}^n$ in the form $\Theta(x) = \xi(x) + \eta(x)$, where $\xi(x)$ and $\eta(x)$ are uncorrelated fields with mean values $E\xi(x) = m(x)$, $E\eta(x) = 0$ and correlation functions $R_1(x_1, x_2) = E\xi(x_1)\xi(x_2)$, $R_2(x_1, x_2) = E\eta(x_1)\eta(x_2)$.

Assume that functions $m(x)$, $R_1(x_1, x_2)$, $R_2(x_1, x_2)$ are unknown and belong to sets $G$, $V_1$, $V_2$ of some functional spaces. Estimates of linear integral operators $S_1m(x) = \int_D l_1(x, y)m(y)dy$, $S_2\xi(x) = \int_D l_2(x, y)\xi(y)dy$ are in the form

$$S_1m(x) = \int_D u_1(x, y)\Theta(y)dy + c_1(y), \quad S_2\xi(x) = \int_D u_2(x, y)\Theta(y)dy + c_2(y),$$

where $D_1$ is measurable by Lebesgue set of space $\mathbb{R}^n$, $l_1(x, y), l_2(x, y)$ are given functions of space $L_2(D_1 \times D_1)$, $c_1(y), c_2(y)$ are functions of space $L_2(D_1)$, $u_1(x, y)$ are functions of space $L_2(D_1 \times D)$. 

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Estimates, for which functions \( \hat{u}_i(x,y), \hat{c}_i(y) \) are found from the condition
\[
\min_{u_i, c_i} \sigma_i(u_i, c_i) = \sigma_i(\hat{u}_i, \hat{c}_i),
\]
where
\[
\sigma_1(u_1, c_1) = \int_{D_1} \sup_{g \in V_1} E[\hat{S}_1 m(x) - \hat{S}_1 m(x)]^2 dx,
\]
\[
\sigma_2(u_2, c_2) = \int_{D_1} \sup_{g \in V_1} E[\hat{S}_2 \xi(x) - \hat{S}_2 \xi(x)]^2 dx,
\]
are called guaranteed linear mean squared estimates of \( S_1 m(x) \) and \( S_2 \xi(x) \).

The existence and uniqueness conditions for such estimates are investigated. If mean value \( m(x) \) belongs to some Sobolev spaces and correlation functions \( R_1(x_1, x_2), R_2(x_1, x_2) \) belong to some sets, it is shown that guaranteed estimates are expressed by generalized solutions of boundary value problems for partial differential equations.

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COX PROCESSES SIMULATION

O. O. Pogoriliak

Simulation of the Cox processes directed by random intensity will be considered.

Let \( \{T, \mathcal{B}, \mu\} \) be a measurable space, \( \mu(T) < \infty \).

**Definition 1** ([1]). Let \( \{Z(t), t \in T\} \), \( T \subset \mathbb{R} \) be a not negative random process. If \( \{\nu(B), B \in \mathcal{B}\} \) under fixed simple function \( Z(t) \) is Poisson process with intensity function \( \mu(B) = \int_B Z(\cdot, t) dt \), that \( \nu(B) \) is said to be a random Cox process driven by process \( Z(t) \).

Let \( Z(t) = \exp\{Y(t)\} \), where \( \{Y(t), t \in T\} \), \( T \subset \mathbb{R} \), be a Brownian motion process, then \( \nu(B) \) is said to be a Cox process directed by the Brownian motion.

Since \( \{\nu(B), B \in \mathcal{B}\} \) is a double stochastic random process, then the model of this process is constructed in two stages. At first we simulate the Brownian motion process \( \{Y(t), t \in T\} \), then we consider some partitioning \( D_T \) of the domain \( T \) and on every element of the partitioning \( D_T \) we construct the model of Poisson random variable with corresponding mean.

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ON ASYMPTOTIC PROPERTIES OF IBRAGIMOV ESTIMATORS IN NONLINEAR REGRESSION MODEL

V. V. Prihodko

Consider the observation model \( X(t) = g(t, \alpha_0) + \varepsilon(t), t \in [0, T] \), where \( g: (-\Delta, \infty) \times A_\gamma \rightarrow \mathbb{R}^1 \) is continuous differentiable function that depends on an unknown parameter \( \alpha_0 \in A \), \( A \subset \mathbb{R}^q \) is bounded open convex set, \( A_\gamma = \bigcup_{\|\varepsilon\| \leq 1} (A + \gamma \varepsilon), \gamma, \Delta > 0 \) are some numbers. Assume that \( \varepsilon(t), t \in \mathbb{R}^1 \) is a measurable stationary Gaussian process with zero mean and smooth spectral density \( f(\lambda, \theta_0) > 0, \lambda \in \mathbb{R}^1, \theta_0 \in \Theta \), where \( \Theta \subset \mathbb{R}^m \) is bounded open convex set, and function \( f(\lambda, \theta) \) defined on \( \mathbb{R}^1 \times \Theta_T, \Theta_T = \bigcup_{\|\varepsilon\| \leq 1} (\Theta + \gamma \varepsilon), \tau > 0 \) is some number. To estimate \( \alpha_0 \) we use the least squares estimator.

Consider a factorization of spectral density \( f(\lambda, \theta) = \sigma^2(\theta) \psi(\lambda, \theta), \lambda \in \mathbb{R}^1, \theta \in \Theta^c \), where \( \sigma^2(\theta) = \int_{-\infty}^{\infty} f(\lambda, \theta) w(\lambda) d\lambda, \int_{-\infty}^{\infty} \psi(\lambda, \theta) w(\lambda) d\lambda = 1 \). Introduce residual periodogram \( I_T(\lambda, \hat{\alpha}_T) = \frac{1}{2\pi T} \int_0^T (X(t) - g(t, \hat{\alpha}_T)) e^{-i\lambda t} dt \), \( \lambda \in \mathbb{R}^1 \), and consider the contrast field \( U_T(\theta, \hat{\alpha}_T) = - \int_{-\infty}^{\infty} I_T(\lambda, \hat{\alpha}_T) w(\lambda) \log \psi(\lambda, \theta) d\lambda, \theta \in \Theta^c \), where \( w(\lambda), \lambda \in \mathbb{R}^1 \) is some even positive weight function. The minimum contrast estimator of \( \theta_0 \in \Theta \), is any random vector such that \( U_T(\hat{\theta}_T, \hat{\alpha}_T) = \min_{\theta \in \Theta^c} U_T(\theta, \hat{\alpha}_T) \).

We prove the consistency and asymptotic normality of \( \hat{\theta}_T \) in the case, when \( \hat{\alpha}_T \) has the same properties. These results extend those of [1].
ASYMPTOTIC NORMALITY OF THE DISCRETIZED MAXIMUM LIKELIHOOD DRIFT PARAMETER ESTIMATOR IN THE HOMOGENEOUS DIFFUSION MODEL

K. Ralchenko

We consider the homogeneous diffusion process given by the stochastic differential equation

\[ X_t = x_0 + \theta \int_0^t a(X_s) ds + \int_0^t b(X_s) dW_s, \]

where \( W_t \) is the standard Wiener process, \( x_0 \in \mathbb{R} \), and \( \theta \) is the unknown parameter. Assume that we observe the process \( X \) at discrete moments of time

\[ t_n^k = \frac{k}{n}, \quad 0 \leq k \leq n^{1+\alpha}, \]

where \( 0 < \alpha < 1 \). In this scheme Mishura [1] proposed the following discretized version of the maximum likelihood estimator

\[ \hat{\theta}_n \approx \frac{\sum_{k=0}^{n^{1+\alpha}} a(X_k^\theta) (X_{k+1}^\theta - X_k^\theta) / b^2 \left( X_k^\theta \right)}{n^{-1} \sum_{k=0}^{n^{1+\alpha}} a^2 \left( X_k^\theta \right) / b^2 \left( X_k^\theta \right)}, \]

and proved its strong consistency. We establish the asymptotic normality of this estimator, assuming the ergodicity of the model.

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LIMIT THEOREMS FOR FUNCTIONALS OF RANDOM FIELDS AND STATISTICAL APPLICATIONS

L. Sakhno

We discuss an approach for establishing limit theorems for functionals of stationary random fields under the integrability conditions on the spectral densities of second and higher orders. Applications for statistical estimation in the spectral domain are presented, including derivation of consistency and asymptotic normality of different classes of parametric estimators.

ON ASYMPTOTIC DISTRIBUTION OF KOENKER-BASSETT ESTIMATOR IN NONLINEAR REGRESSION MODEL

I. N. Savych

Nonlinear regression model \( X(t) = g(t, \theta) + \varepsilon(t), t \geq 0 \), is considered where \( g(t, \theta) \in C([0, +\infty) \times \Theta^d), \Theta \subset \mathbb{R}^d \), is an open bounded set, \( \varepsilon(t) = G(\xi(t)), t \in \mathbb{R}, \) with Borel function \( G(x), x \in \mathbb{R}, \mathbb{E}\varepsilon(0) = 0, \mathbb{E} \varepsilon^2(0) < \infty \). Random process \( \xi(t), t \in \mathbb{R}, \) is measurable stationary Gaussian process, \( \mathbb{E}\xi(0) = 0, \mathbb{E}\xi(t)\xi(0) = B(t) = \sum_{j=0}^r A_j B_{\alpha_j \chi_j}(t), t \in \mathbb{R}, r > 0, \) where \( A_j > 0, \sum_{j=1}^r A_j = 1, B_{\alpha_j \chi_j}(t) = \frac{\cos(\chi_j t)}{(1+\chi_j^2)^{\frac{1}{2}}}, j = 0, r, 0 = \chi_0 < \chi_1 < \ldots < \chi_r < +\infty, \alpha_j \in (0, 1). \)

For distribution function \( F(x) \) of \( \varepsilon(0) \) put \( F(0) = \beta \in (0, 1), p(x) = F'(x), p(0) > 0 \). Consider Koenker-Bassett estimator \( \hat{\theta}_T \) of \( \theta \in \Theta, \) that is \( M \)-estimator defined by loss function \( \rho_\beta(x) = \beta x \chi_{[0,\infty)}(x) + (\beta - 1)x \chi_{(-\infty,0]}(x). \)

Let \( \mu \) be spectral measure of regression function \( g(t, \theta), \) such that spectral density \( f(\lambda), \lambda \in \mathbb{R}, \) of \( \xi(t) \) is \( \mu \)-admissible. A function \( \Psi(x) = \beta - \chi (G(x) < 0), x \in \mathbb{R}, \) has Hermite rank \( m \geq 1. \)
Denote by \( d^2_T(\theta) = \int_0^T \left( g_{ij}(t, \theta) \right)^2 dt, \quad d^2_T(\theta) = \text{diag} \left( d^2_T(\theta) \right)^T \). The normed estimator \( d_T(\theta) \) is asymptotically normal (see, [1,2]) \( N(0, D) \), \( D = \frac{2\pi}{M} \Lambda \left( \sum_{j=m}^{\infty} \frac{C^2_j(\Psi)}{\mu(\lambda, \theta)} \int_\Lambda f^*(\lambda) \mu(d\lambda, \theta) \right) \Lambda \), where \( \Lambda = \left( \int_\Lambda \mu(d\lambda, \theta)^{-1} \right)^{-1} \), \( f^*(\lambda) \) is the \( j \)-th convolution of spectral density \( f(\lambda) \), \( C_j(\Psi) = \int_\Lambda \Psi(x) H_j(x) \varphi(x) dx, \quad H_j(x), j = 0, 1, \ldots \), are Hermite polynomials, \( \varphi(x) = e^{-\frac{x^2}{2}}/\sqrt{2\pi}, x \in \mathbb{R} \).

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ON SOME METHODS OF FINDING OPTIMAL ESTIMATES IN STATISTICS OF RANDOM PROCESSES

A. D. Shatashvili1, T. A. Fomina2

Let \( \{\Omega, \varphi, P\} \) be a fixed probability space, and \( H \) be a real separable Hilbert space with scalar product \((x, y)\) and norm \( \|x\| \), \( x, y \in H \). Denote by \( L_2 = L_2\{[0, a], H\} \) the Hilbert space of functions, defined on the interval \([0, a]\), with values in \( H \), and square integrable in the norm \( H \). In the space \( L_2 \) consider a system of nonlinear and linear evolution differential equations

\[
\begin{align*}
\frac{dy(t)}{dt} - A(t)y(t) + A_1(t)y(t) + a f(t, y(t)) &= \xi(t) \\
0 \leq t \leq a, \quad y(0) &= \xi(0) = 0 \quad \text{(mod P)}
\end{align*}
\]

(1)

\[
\begin{align*}
\frac{dx(t)}{dt} - A(t)x(t) + A_1(t)x(t) &= \xi(t) \\
0 \leq t \leq a, \quad x(0) &= \xi(0) = 0 \quad \text{(mod P)}
\end{align*}
\]

(2)

where \( a \) is some parameter, \( \xi(t) \) is a Gaussian process defined on the interval \([0, a]\), with values \( \xi(t) \in H \), with zero expectation and correlation operator \( R_\xi(t, s) = f(t, y(t)) \) is a non-linear function, defined on \([0, a] \times H\), with values in \( H \), square integrable in the norm \( H \) with respect to \( t \) for all \( y(t) \in H \), and differentiable with respect to \( y \); operators \( A(t) \) and \( A_1(t) \) are unbounded linear operators.

In the talk we consider the situation when the perturbing process in (1), (2) is a stationary Gaussian process with known eigenvalues and eigenfunctions for the correlation operator. In this case the form of the Radon-Nikodym density \( \frac{d\pi}{d\mu} \) allows to obtain the explicit expression of optimal estimates for the solution \( y(t) \) to (1) in extrapolation and filtration problems. Further, in some particular cases we can obtain an optimal estimate for the solution to the boundary value problem.

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STATISTICAL ESTIMATORS OF TWO STRAIGHT LINES BY OBSERVATIONS OF POINTS WITH RANDOM PERTURBATIONS

S. V. Shklyar

Consider a problem of fitting a set of points with straight lines. “True points” are supposed to lie exactly on the lines. The points are observed with Gaussian errors. Their covariance matrix is proportional to a unit matrix and is known up to a scalar factor. Parameters of the lines are to be estimated. This problem is important in cluster analysis.

Different estimators can be constructed by the following methods: Hough transform, orthogonal regression and different versions of maximum likelihood method, method of moments.

We focus on fitting with two lines. At first the points are fitted with conic section as described in Shklyar et al. (2007), and if the conic is a hyperbola, its asymptotes are treated as the estimates of the lines. Sufficient conditions for the consistency and asymptotic normality of the estimators are provided. The estimates are compared numerically with ones obtained by other methods. For fitting with three or more lines, the estimators can be constructed in a similar way.

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ADAPTIVE TEST FOR MEANS HOMOGENEITY IN MIXTURE MODELS

O. V. Sugakova

Let the observed sample \( X_n = (\xi_{1:n},...,\xi_{n:n}) \) be sampled from a mixture with varying concentrations: \( \xi_{j:n} \) are independent random vectors with distribution

\[
P(\xi_{n:n} \in A) = \sum_{i=1}^{M} p_{j:n} F_i(A),
\]

where \( F_i \) is the unknown distribution of \( i \)-th mixture component; \( p_{j:n} \) is the known probability to observe a subject from \( i \)-th component at \( j \)-th observation.

We consider the hypothesis \( H_0 : \bar{g}_i = \bar{g}_2 \), where \( \bar{g}_i = \int_{-\infty}^{\infty} g(x) F_i(dx) \) is the functional moment of \( i \)-th component with the moment function \( g \). The test is based on a statistics \( S_n(b) = \frac{1}{n} \sum_{j=1}^{n} b_{j:n} g(\xi_{j:n}) \), where \( b_{j:n} \) are some weights, such that \( ES_n(b) = 0 \) under \( H_0 \). For non-adaptive tests \( b_{j:n} \) are non-random, dependent on \( (p_{j:n})_{j=1,...,n;i=1,...,M} \) only. It is shown, that the normalized statistics \( T_n(b) = \frac{S_n(b)}{\sqrt{\text{Var}(S_n(b))}} \) is asymptotically \( N(0,1) \) under \( H_0 \), where \( \hat{V}_n(b) \) is an estimate for \( \text{Var}S_n(b) \). Than the power of test based on \( T_n(b) \) is investigated on local alternatives and weights vector \( b^* \) is derived which yields the most asymptotically powerful test. Since \( b^* \) depend on unknown parameters, it is replaced by it’s estimate \( \hat{b} \) in the adaptive test. The adaptive test based on \( T_n(\hat{b}) \) possess the same asymptotic power as the best non-adaptive test.

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CONVERGENCE OF ESTIMATORS IN A POLYNOMIAL FUNCTIONAL MODEL UNDER MEASUREMENT ERRORS

Ya. Tsaregorodtsev\(^1\), A. Kukush\(^2\)

A polynomial functional measurement error model is considered,

\[
y_i = \beta_0 + \beta_1 \xi_i + \beta_2 \xi_i^2 + \cdots + \beta_k \xi_i^k + \varepsilon_i, \\
x_i = \xi_i + \delta_i, \quad i = 1, \ldots, n.
\]

The couples \((y_i, x_i), i = 1, \ldots, n\), are observed. The unobservable latent variable \( \xi \) is nonrandom. The variance of the measurement error \( \delta_i \) and its covariance with the error \( \varepsilon_i \) are assumed known, as well as some error moments.

The adjusted least squares (ALS) estimator of regression parameters \( \beta_0, \ldots, \beta_k \) adapts an ordinary least squares estimator for the measurement errors. Using the Rosenthal moment inequality, we present sufficient conditions for the weak and strong consistency (as the sample size grows) of the estimator that are lighter compared with Cheng and Schneeweiss [1], as well as conditions for the asymptotic normality of the estimator. Under a bit stronger assumptions, the estimator of the asymptotic covariance matrix is consistent as well.

Cheng et al. [2] introduce a modified ALS estimator that is more stable for small and moderate sample. We discuss the applicability of the modification and give conditions for its consistency.

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Identification of linear systems is a subject of much interest both in theory and in applications. There is a wide range of methods using time and frequency domain data. We focus on a model related to partially observed linear systems in a continuous time domain, where the function we would like to control is not observed directly. The objective is to perform estimation of different functional characteristics of the underlying model.

We assume that a process \( X_t = (X_t, 0 \leq t \leq T) \) is observed, and this process satisfies the following system of stochastic differential equations:

\[
\begin{align*}
  dX_t &= h_t Y_t dt + \varepsilon \, dW_t, \quad X_0 = 0, \\
  dY_t &= g_t Y_t dt + \varepsilon \, dV_t, \quad Y_0 = y_0 \neq 0, \quad 0 \leq t \leq T.
\end{align*}
\]

Here, \( W_t \) and \( V_t \), \( 0 \leq t \leq T \), are two independent Wiener processes. The process \( Y_t = (Y_t, 0 \leq t \leq T) \) cannot be observed directly, but it is the one we would like to control.

In this model, we consider the problem of estimation of different functions on \( 0 \leq t \leq T \), in the asymptotics of a small noise, i.e., as \( \varepsilon \to 0 \). We propose kernel-type estimators for the functions \( f_t := h_t y_t, h_t, y_t, g_t, 0 \leq t \leq T \), and study their properties. The notation \( y_t, 0 \leq t \leq T \), stands for the solution of the deterministic version of the above model, that is, the one where the noise terms are dropped.

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**ABOUT THE CONSISTENT CRITERIA FOR CHECKING HYPOTHESES**

Z. S. Zerakidze\(^1\), M. O. Mumladze\(^2\), J. K. Kiria\(^3\), T. V. Kiria\(^4\)

In this note, without using the axiom of choice we construct an example of strongly separable statistical structure which does not admit a consistent criteria for hypotheses testing.

Let \((E, S)\) be a measurable space with a given family of probability measures: \(\{\mu_i, i \in I\}\). The following definitions come from works \([1,2]\).

**Definition 1.** We call an object \(\{E, S, \mu_i, i \in I\}\) the statistical structure.

**Definition 2.** A statistical structure \(\{E, S, \mu_i, i \in I\}\) is called strongly separable if there exists a family of pairwize disjoint \(S\)-measurable subsets \(X_i, i \in I\) in \(E\) such \(\mu_i(X_i) = 1\) for all \(i \in I\).

**Definition 3.** Any assumption defining the form of the unknown distribution function is called hypotheses.

Let \(H\) be the set of parameter and \(B(H)\) the \(\sigma\)-algebra of subsets of \(H\) which contains all finite subsets of \(H\).

**Definition 4.** A statistical structure \(\{E, S, \mu_i, i \in I\}\) will be said to admit consistent criteria of hypotheses testing if there exists a measurable map \(\delta : (E, S) \to (H, B(H))\), such that \(\mu_h(\{x|\delta(x) = h\}) = 1\) for all \(h \in H\).

Denote by \(w_1\) the first uncountable ordinal number. The interval \([0, w_1)\) consists of no more than countable ordinals. Let equip this interval with the order topology (cf. [3]).

Let us denote by \(B([0, w_1])\) the Borel \(\sigma\)-algebra of \([0, w_1)\) generated by the order topology. Let us define the probability measure on \(B([0, w_1])\) as follows

\[
\mu(Z) = \begin{cases} 
1, & \text{if } Z \text{ contains unbounded closed set,} \\
0, & \text{if } [0, w_1] \setminus Z \text{ contains unbounded closed set.}
\end{cases}
\]

Let \(E = [0, w_1) \times [0, w_1)\), and \(S = B([0, w_1) \times [0, w_1))\), \(H = (\{0\} \times [0, w_1)) \cup ([0, w_1) \times \{0\})\). For each \(h \in H\) we set

\[
\mu_h = \begin{cases} 
\mu(P_{R_1}(Z \cap ([0, w_1) \times \{\xi\}))), & \text{if } h \in (0, \xi), \xi \in w_1, \\
\mu(P_{R_2}(Z \cap ([\xi] \times [0, w_1])), & \text{if } h \in (0, \xi), \xi \in w_1,
\end{cases}
\]

where \(Z \in S\). The main result is formulated as follows.

**Theorem 5.** An object \(\{E, S, \mu_i, i \in I\}\) is an example of a strongly separable statistical structure which does not admit a consistent criteria for hypotheses testing.

**References**


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ON PERIODOGRAM ESTIMATORS OF THE ALMOST PERIODIC SIGNAL PARAMETERS

B. M. Zhurakovskiy

Suppose the random process \( X(t) = A_0 \varphi(\omega_0 t) + \varepsilon(t), t \in [0, T] \), is observed, where \( \varphi(t) = \sum_{k=\infty}^{+\infty} a_k e^{i \lambda_k t}, t \in \mathbb{R}^1 \), and \( \lambda_0 = 0, \lambda_k > 0, \) with \( k > 0, a_{-k} = 0 \), \( \lambda_{-k} = -\lambda_k, |\lambda_l - \lambda_k| \geq \Delta > 0 \) for \( l \neq k \) with some fixed \( \Delta > 0 \), \( \sum_{k=-\infty}^{+\infty} |a_k| < \infty \). Assume also that \( A_0 > 0, \omega_0 \in (\omega, \overline{\omega}), 0 < \omega < \overline{\omega} < \infty \), and there exists \( i_0 > 0 \) such that \( |a_{i_0}| > |a_l|, l \neq \pm i_0, \varepsilon(t), t \in \mathbb{R}^1 \), is a local functional of a Gaussian stationary process \( \xi \), that is \( \varepsilon(t) = G(\xi(t)), \xi(x), x \in \mathbb{R}^1 \), is a Borel function such that \( \mathbb{E}{\varepsilon(0)} = 0, \mathbb{E}{\varepsilon^2(0)} < \infty \). The random process \( \xi(t), t \in \mathbb{R}^1 \), is a measurable stationary Gaussian process \( \mathbb{E}{\xi(0)} = 0 \) with covariance function \( B(t) = \mathbb{E}{[\xi(0)\xi(t)]} = \sum_{j=0}^{\infty} D_j B_{\alpha_j, \psi_j}(t), t \in \mathbb{R}^1, k \geq 0, \sum_{j=0}^{k} D_j = 1, D_j \geq 0, j = 0, \overline{\omega} \), where \( B_{\alpha_j, \psi_j}(t) = \frac{\cos(\psi_j t)}{(1+\overline{\omega}^2)^{\alpha_j/2}}, 0 \leq \psi_0 < \psi_1 < ... < \psi_k, \alpha_j > 0, j = 0, \overline{\omega} \). The periodogram estimator of the frequency \( \omega_0 \) is said to be any random variable \( \omega_T \in [\lambda_{i_0}, \lambda_{i_0}], 0 < \lambda_{i_0} < \overline{\lambda}_{i_0} \) such that \( Q_T(\omega_T) = \max_{\omega \in [\lambda_{i_0}, \lambda_{i_0}]} Q_T(\varphi) \). We define the periodogram estimator of an amplitude \( A_0 \) as \( A_T = \frac{1}{\max_{\omega \in [\lambda_{i_0}, \lambda_{i_0}]} Q_T(\varphi)} \). Under some additional conditions on \( G \) it is proved that the vector \( (T^{1/2}(A_T - A_0), T^{3/2}(\omega_T - \lambda_{i_0} \omega_0)) \) is asymptotically as \( T \to \infty \) normal with zero mean and covariance matrix

\[
2\pi |a_{i_0}|^{-2} \sum_{j=0}^{\infty} C_j^2(G) \int f^{*j}(\lambda_{i_0} \omega_0) \begin{pmatrix} 1/2 & 0 \\ 0 & 6A_0^{-2} \end{pmatrix},
\]

where \( f^{*j}(\lambda) = \int_{-\pi}^{\pi} f(\lambda - \lambda_2 - \cdots - \lambda_j) \Pi_{i=2}^{j} f(\lambda_i) d\lambda_2 \cdots d\lambda_j \), is the \( j \)-th convolution of the spectral density \( f(\lambda) \) of the random process \( \xi \), \( m \) is Hermite rank of \( G \) and \( C_j(G) = \int_{-\infty}^{\infty} G(x)H_k(x)\varphi(x)dx, k \geq 0, \varphi(x) = (2\pi)^{-1/2}e^{-x^2/2}, H_k(x) \) are Hermite polynomials. This statement generalizes the result of the paper [1].

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ON A SIMULATION METHOD FOR MODEL CHOICE

S. Zwan zig

A data set consisting of observations of a response variable and of measurements of a number of covariables is given. We search for a model explaining the behavior of the response variable which includes only essential covariables.

The proposed simulation method tests the influence of stepwise perturbation of covariables on the quality of model fit. The main idea is that the disturbance of an unimportant covariable will have no effect on the model fit. Least Squares as well as Ridge and Lasso are studied as criteria of the model fit. The method works well for measurement error models.

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STOCHASTIC ANALYSIS AND STOCHASTIC DIFFERENTIAL EQUATIONS, PARTIAL STOCHASTIC DIFFERENTIAL EQUATIONS

DIFFERENTIABILITY OF STOCHASTIC FLOWS FOR SDE’S WITH NONREGULAR DRIFTS

O. V. Aryasova¹, A. Yu. Pilipenko²

Consider the d-dimensional stochastic differential equation (SDE)

\[
\begin{aligned}
d\varphi_t(x) &= a(t, \varphi_t(x))dt + \sum_{k=1}^{m} \sigma_k(t, \varphi_t(x))dw_k(t), \\
\varphi_0(x) &= x,
\end{aligned}
\]

where \(x \in \mathbb{R}^d, d \geq 1, m \geq 1, (w(t))_{t \geq 0} = (w_1(t), \ldots, w_m(t))_{t \geq 0}\) is a standard \(m\)-dimensional Wiener process, the drift coefficient \(a : [0, \infty) \times \mathbb{R}^d \rightarrow \mathbb{R}^d\) and the diffusion coefficient \(\sigma : [0, \infty) \times \mathbb{R}^d \rightarrow \mathbb{R}^d \times \mathbb{R}^m\) are Borel measurable and bounded. We assume that \(\sigma \in W_{2d+2,\text{loc}}([0, \infty) \times \mathbb{R}^d)\) and \(\sigma\) is uniformly elliptic. Under these assumptions on the coefficients there exists a unique strong solution to equation (1) (see [1]).

By analogy with Kato’s class we define the class \(K\) of measures on \([0, \infty) \times \mathbb{R}^d\) such that

\[
\lim_{t \downarrow 0} \sup_{t_0 \in [0, \infty)} \int_{R^d} \int_{t_0}^{t_0+t} \frac{1}{(2\pi(s-t_0))^{d/2}} \exp \left\{-\frac{|y-x|^2}{2(s-t_0)}\right\} \nu(ds, dy) = 0.
\]

We say that a signed measure \(\nu\) belongs to the class \(K\) if \(\nu^+\) and \(\nu^-\) are of the class \(K\), where \(\nu = \nu^+ - \nu^-\) is the Hahn–Jordan decomposition of \(\nu\).

Let for each \(1 \leq i \leq d\), \(a^i(t, \cdot)\) be a function of bounded variation on \(\mathbb{R}^d\). Assume that for all \(1 \leq i, j \leq d\), \(1 \leq k \leq m\) the signed measures \(\nu^{ij}(dt, dy) := \frac{\partial a^i}{\partial y_j}(t, dy)dt\) and \(\left(\frac{\partial a^i}{\partial y_j}(s, y)\right)^{\rho+2} dsdy\) for some \(\rho > 0\) belong to the class \(K\).

We obtain a representation for the derivative \(\nabla_x \varphi_t(x)\) in terms of intrinsic parameters of the initial equations. The theory of additive functionals is used.

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EVALUATION OF THE FISHER INFORMATION FOR DISCRETELY OBSERVED SDE’S WITH LÉVY NOISE

S. V. Bodnarchuk

Consider a stochastic differential equation

\[
dx(t) = a_b(X(t))dt + dZ(t),
\]

where \(Z\) is one-dimensional Lévy process without diffusion component, \(a : \Theta \times \mathbb{R} \rightarrow \mathbb{R}\) is a measurable function, \(\Theta \subset \mathbb{R}\) is a parametric set.

Our goal is to calculate the Fisher information for sample of discretely observed values of solution of SDE (1). In the report we consider an example of such calculation.

This report is based on the joint research with Kulik A. M. and Ivanenko D. O.

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RANDOM EVOLUTION WITH CONTROL AND MARKOV SWITCHING

Ya. M. Chabanyuk

Asymptotic behaviour of random evolutions with Markov switching in averaging, diffusion approximation, Poisson approximation and Lévy approximation schemes was considered in [1]. In [2] results for such behaviour were obtained for procedure of stochastic approximation (SAP) and optimization (SOP) as controlled Markov process.

In this work SAP with Markov switching in diffusion approximation scheme and impulse perturbation with independent increments was considered. Sufficient conditions of convergence of random evolution with control in asymptotically small diffusion scheme were obtained.

Results for SOP for the project size index "s" the prediction of the software reability when testing for errors with the function of the intensity error detection [3].

\[ \lambda(s,t) = \hat{\alpha} \beta^{s+1} t^{s} \exp(-\beta t), \]

where \( \hat{\alpha} \) and \( \hat{\beta} \) obtain by maximum likelihood method.

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RECENT DEVELOPMENTS IN RANDOM AFFINE RECURSIONS

E. Damek

We consider the following affine recursion in \( \mathbb{R}^d \)

\[ X_n = A_n X_{n-1} + B_n, \quad n \geq 1 \]  

(1)

where \((A_n, B_n)\) is a sequence of i.i.d. (independent identically distributed) random variables with values in \( GL(\mathbb{R}^d) \times \mathbb{R}^d \) and \( X_0 \in \mathbb{R}^d \) is the initial distribution. The generic element of the sequence \((A_n, B_n)\) will be denoted by \((A, B)\). Under mild contractivity hypotheses the sequence \( X_n \) converges in law to a random variable \( R \), which is the unique solution of the stochastic difference equation

\[ R = A R + B, \quad R \text{ independent of } (A, B) \]  

(2)

and equality is meant in law. The main issues concerning (1) are characterization of the tail of \( R \), regularity of the law of \( R \), behavior of iterations \( X_n \).

First results were obtained already in seventies by Kesten for matrices with positive entries and by Grincevicius and Vervaat in the one dimensional case. However recently, due to its importance, equation (2) has again attracted attention of many people the contribution of the Wrocław team being essential.

With so called Kesten assumptions \( R \) has a heavy tail behavior which, means that there is \( \alpha > 0 \) such that

\[ \lim_{t \to \infty} \mathbb{P}(|R| > t)^{\alpha} = C_\alpha > 0. \]

However, there is still no satisfactory description of the constant \( C_\infty \) as well as the rate of convergence of \( \mathbb{P}(||R|| > t)^{\alpha} \) to \( C_\infty \). I am going to talk about the latter problem both in the case of recursion (1) and the Lipschitz iterative systems modeled on it. Good formulae for \( C_\infty \) are important from the point of view of applications.

The talk is based on the joint work with Dariusz Buraczewski, Jacek Zienkiewicz – Wrocław University, Rafał Latala, Piotr Nayar – Warsaw University and Tomasz Tkocz – University of Warwick.

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\( L^2 \)-RATES OF APPROXIMATION OF NONSMOOTH INTEGRAL-TYPE FUNCTIONALS

Iu. V. Ganychenko

We consider an \( \mathbb{R}^d \)-valued Markov process \( X_t, t \geq 0 \) and establish strong \( L^2 \)-rates of approximation of integral-type functional \( I_T \) of such a process by integral sums \( I_{T,n} \), where

\[ I_T(h) = \int_0^T h(X_t) dt, \quad I_{T,n}(h) = \frac{T}{n} \sum_{k=0}^{n-1} h(X_{(kT)/n}), \quad n \geq 1. \]
We have developed two methods of obtaining the stochastic integral representation of nonsmooth (in Malliavin sense) Brownian functionals and have found explicit form of integrands in this representations. The first method demands smoothness only for conditional mathematical expectation of the considered functional, instead of the usual requirement of smoothness of the functional (as it was in the well-known Clark-Ocone formula). The second method is based on the notion of semimartingale local time and the well-known theorem of Trotter-Mayer which establishes the connection between a predictable square variation of a semimartingale and its local time (see [1]).

The offered methods allow us obtain the integral representations for the indicator functionals and other nonsmooth functionals. Also we have discussed the martingale representation formula obtained by Cont and Fournier within the functional Itô calculus (see [2]).

**Theorem 1.** Let $F = (B_T - K)^+ I_{[B_T \leq L]}$ (where $B_T = \max_{0 \leq t \leq T} B_t$). Then the following stochastic integral representation is fulfilled

$$F = EF + \int_0^T \left[ \Phi \left( \frac{B_t - K}{\sqrt{T - t}} \right) - \Phi \left( \frac{B_t - 2L + K}{\sqrt{T - t}} \right) - \frac{2L - K}{\sqrt{T - t}} \varphi \left( \frac{L - B_t}{\sqrt{T - t}} \right) \right] dB_t$$

(where $K$ and $L > 0$ are constants, $\Phi$ is the standard normal distribution function and $\varphi$ is its density function).

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**CAUCHY REPRESENTATION FORMULA FOR SOLUTION OF A SYSTEM OF THE LINEAR NONHOMOGENEOUS SDE WITH SKOROHOD INTEGRAL**

O. V. Ilchenko

One-dimension case of this problem was considered in [3] by means of Girsanov’s theorem approach. In higher dimensions it needs the Wick product techniques.

Denote by $\int_0^T u_s \, \delta w_s$ and $\int_0^T u_s \, dw_s$ the Skorohod integral and the Ito integral respectively [2]; $\ast$ is the Wick product; $M^d$ is the space of $d \times d$ matrices. For $F$ with Ito-Wiener expansion $F = \sum_{n=1}^{\infty} I_n(f_n)$ denote $H_0^+ = \bigcup_{\lambda \in \Lambda} H_\lambda$; $H_\lambda = \{ F : ||F||_2 < \infty \}; \ ||F||_2 = \sum_{n=1}^{\infty} n! \lambda^n ||f_n||_2^2, \ 0 < \lambda < +\infty$ [1].

Consider the following systems of stochastic differential equations

$$x_t = x_0 + \int_0^t (A_s x_s + \varphi_s) \, ds + \int_0^t (B_s x_s + \psi_s) \, \delta w_s;$$

$$H_0^t = I + \int_0^t A_u H_0^u \, du + \int_s^t B_u H_0^u \, dw_u.$$
Theorem 1. Let $A \in L^1([0,T];M^d)$, $B \in L^2([0,T];M^d)$, $x_0 \in H_{0+}(\mathbb{R}^d)$, $\varphi$, $\psi \in H_{0+}(L^2[0,T];\mathbb{R}^d)$. The solution to (1) $x_t \in H_{0+}(L^2[0,T];\mathbb{R}^d)$ and have the form

$$x_t = H_0^\varphi \circ \left( x_0 + \int_0^t (H_0^\varphi)^{(-1)} \circ \varphi_s ds + \int_0^t (H_0^\psi)^{(-1)} \circ \psi_s \delta w_s \right).$$

In addition, $x_t \in H_\infty(L^2[0,T];\mathbb{R}^d)$ if $x_0 \in H_\infty(\mathbb{R}^d)$ and $\varphi$, $\psi, \varphi, \psi \in H_\infty(L^2[0,T];\mathbb{R}^d)$.

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**ON THE SKOROHOD EMBEDDING PROBLEM AND FBSDE**

Peter Imkeller

A link between martingale representation and solutions of the Skorokhod embedding problem has been established by R. Bass. A generalization of his approach to systems of forward-backward stochastic differential equations leads us to solutions of the Skorokhod embedding problem for certain diffusion processes.

The talk is based on joint work with Alexander Fromm and David Prömel.

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**ON FELLER SEMIGROUPS ASSOCIATED WITH ONE-DIMENSIONAL INHOMOGENEOUS DIFFUSION PROCESSES WITH MEMBRANES**

B. I. Kopytko¹, R. V. Shevchuk²

Using methods of classical potential theory we obtain the integral representation of two-parameter operator semigroup associated with Feller process on a line that is a result of pasting together at fixed points the finite number of diffusion processes given by their generating differential operators. The behavior of the required process at each point of pasting together where membranes are placed is described by different variants of general Feller-Wentzell type conjugation condition [1], which obligatory contains nonlocal term, i.e., integral term. We also investigate some properties of the constructed process. The results represented here generalize the similar results we obtained earlier in [2] where the case of membrane placed only at one fixed point was considered.

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**PARAMETRIX METHOD, UNIQUENESS, AND NON-UNIQUENESS OF WEAK SOLUTION TO AN SDE WITH STABLE NOISE**

A. M. Kulik

For an SDE in $\mathbb{R}^d$

$$dX_t = a(X_t)dt + \sigma(X_t-)dZ_t^\alpha$$

(1)

driven by a symmetric $\alpha$-stable process $Z^\alpha$, existence and uniqueness of the weak solution is proved under the following assumptions:

- coefficient $\sigma$ satisfies $\sigma^\alpha > 0$ and is Hölder continuous with some positive Hölder index;
- coefficient $a$ is Hölder continuous with the Hölder index $\gamma$ such that

$$\alpha + \gamma > 1.$$  

(2)
Explicit upper and lower estimates for the transition probability density of the solution $X$, considered as a Markov process, are provided, as well. The proofs are analytical and based on a specially designed version of the parametrix method, which is suitable for studying fundamental solutions of gradient perturbations of $\alpha$-stable generators without the assumption $\alpha > 1$, which is typically used in the available literature. The case $\alpha < 1$ exhibits substantially new effects, when compared either with the classical diffusion case or the case $\alpha \in (1,2)$. For instance, an examples are available where $\alpha + \gamma = 1 - \varepsilon$ with small $\varepsilon > 0$, but SDE (1) fails to have the uniqueness property. This, in particular, shows that the condition (2) is close to the necessary one.

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ASYMPTOTIC BEHAVIOR OF THE INTEGRAL FUNCTIONALS FOR UNSTABLE SOLUTIONS OF ONE-DIMENSIONAL ITO STOCHASTIC DIFFERENTIAL EQUATIONS

G. L. Kulinich$^1$, S. V. Kushnirenko$^2$, Yu. S. Mishura$^3$

We consider Itô stochastic differential equation $d\xi(t) = a(\xi(t))dt + dW(t)$, where $a$ is measurable, bounded, real function and satisfies conditions supplying the unstable property of the unique strong solution $\xi$. The explicit form of normalizing factor for functionals $\int_0^t g(\xi(s))d\xi(s)$, where function $g$ is real and locally square integrable is established to provide the weak convergence to the limiting process. The asymptotic behavior as $t \to \infty$ of functionals $\int_0^t g(\xi(s))ds$ was investigated in [1]. The paper [2] contains similar result for the functionals $\int_0^t g(\xi(s))dW(s)$.

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PARABOLIC EQUATIONS WITH RANDOM BOUNDARY CONDITIONS

I. V. Marina

Consider a boundary value problem for a parabolic equation:

$$
\frac{\partial}{\partial x} \left( p(x) \frac{\partial z(t,x)}{\partial x} \right) - q(x) z(t,x) - \rho(x) \frac{\partial z(t,x)}{\partial t} = 0,
$$

(1)

$$
0 < x < l, 0 < t < T < \infty,
\frac{\partial z(0,x)}{\partial x} = 0, 0 \leq x \leq l,
$$

(2)

$$
z_x(t,0) - \alpha z(t,0) = \eta_1(t), z_x(t,l) + \alpha z(t,l) = \eta_2(t),
$$

(3)

where $\eta_1(t), \eta_2(t), 0 \leq t < T < \infty$, are independent stochastic processes belonging to the Orlicz space $L_U(\Omega)$. Then solution of the problem (1)-(3) has the form:

$$
z(t,x) = \left( \frac{1}{2l + \alpha l^2} \eta_2(t) - \frac{1 + \alpha t}{2l + \alpha l^2} \eta_1(t) \right) x^2 + \eta_1(t) x + \sum_{n=1}^{\infty} X_n(x) \int_0^t \gamma_n(t') e^{-\lambda_n(t-t')}dt'.
$$

We study conditions justifying the application of the Fourier method for parabolic equations with random boundary conditions and obtain bounds for the distribution of the supremum of solutions of these equations.

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Let \((\xi(t), t \geq 0)\) be a solution to stochastic differential equation
\[ d\xi(t) = a(\xi(t))dt + dw(t), \ t \geq 0, \]
where \(a(\cdot)\) is a continuous function such that
\[ a(x) = c_+/x \text{ for } x > 1 \quad \text{and} \quad a(x) = c_-/x \text{ for } x < -1. \]
Assume that \(c_+ > -1/2\) and consider a sequence of processes
\[ (\xi_n(t) = \frac{1}{\sqrt{n}}(\xi(nt)), \ t \geq 0), \ n \geq 1. \]
It was shown in [1,2] that the sequence \(\{\xi_n(\cdot)\}\) of absolute values of the processes converges weakly as \(n \to \infty\) to a Bessel process.

We find the weak limit of the sequence \(\{\xi_n(\cdot)\}\). For example, if \(c_+ = c_- \in (\frac{1}{2}, \frac{1}{2})\), then the limit process is a skew Bessel process.

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**INTEGRAL EQUATIONS WITH A GENERAL STOCHASTIC MEASURE**

V. M. Radchenko

Let paths of random function \(h(t)\) belong to Besov space \(B^{\alpha}_{2\alpha}(a, b)\), \(1/2 < \alpha < 1\), \(\mu\) be a general stochastic measure,
\[ d\nu_n = a + k2^{-n}(b - a), \quad 0 \leq k \leq 2^n, \quad \Delta_n = (d(k-1)n, d(kn)], \quad 1 \leq k \leq 2^n. \]
We define
\[ \int_{(a, b)} h \, d\mu := \lim_{n \to \infty} \sum_{k=1}^{2^n} h(d(k-1)n, \omega)\mu(\Delta_n) \quad \text{a. s.} \]
Consider equation
\[ u_\varepsilon(x) = g(x) + \varepsilon \int_{(a, b)} f(x, y)u_\varepsilon(y) \, d\mu(y), \quad (1) \]
where \(u_\varepsilon(x)\) is an unknown random function.

**Theorem 1.** Let paths of random function \(g(t)\) belong to \(B^{\alpha}_{2\alpha}(a, b)\), \(1/2 < \alpha < 1\), and random function \(f(x, y, \omega) : [a, b]^2 \times \Omega \to \mathbb{R}\) be such that
\[ |f(x_1, y_1) - f(x_2, y_1)| \leq K_1(\omega)|x_1 - x_2|^\delta_x, \quad \delta_x > 0, \]
\[ |f(x_1, y_1) - f(x_1, y_2)| \leq K_2(\omega)|y_1 - y_2|^\delta_y, \quad \delta_y > 0, \]
\[ |f(x_1, y_1) - f(x_1, y_2) - f(x_2, y_1) + f(x_2, y_2)| \leq K_{xy}(\omega)|x_1 - x_2|^{\delta_x} |y_1 - y_2|^{\delta_y}. \]
Then equation (1) has a unique solution on some \(\Omega_\varepsilon \subset \Omega\), and \(P(\Omega_\varepsilon) \to 1, \varepsilon \to 0. \)

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PATH-DEPENDENT INFINITE-DIMENSIONAL SDE: AN ENTROPY APPROACH

S. Röelly

We present some recent existence (and uniqueness) results on weak solutions of infinite-dimensional stochastic differential equations driven by a Brownian term. The drift function is very general, in the sense that it is path-dependent and non-regular. The originality of our method leads in the use of the specific entropy as a tightness tool and on a description of such stochastic differential equation as solution of a variational problem on the path space.

The talk is based on joint works with P. Dai Pra and D. Dereudre.

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CONTINUOUS DEPENDENCE OF SOLUTIONS TO SDE’S DRIVEN BY FRACTIONAL BROWNIAN MOTION ON THE HURST INDEX OF A DRIVING SIGNAL

T. Shalaiko

We consider the following \( d \)-dimensional stochastic differential equation:

\[
dX^H_t = a(X^H_t)dt + b(X^H_t)dB^H_t, \tag{1}
\]

where \( a: \mathbb{R}^d \to \mathbb{R}^d \), \( \sigma: \mathbb{R}^d \to \mathbb{R}^{d \times m} \) and \( B^H = \{ B^H_t, t \in \mathbb{R} \} \) is an \( m \)-dimensional fractional Brownian motion with Hurst index \( H \in (0, 1) \). Results from the rough path analysis imply that the law of \( X^H \) depends continuously on \( H \), e.g., \( P_{X^H} \to P_{X^{H_0}} \), when \( H \to H_0 \) in case of \( H \geq 1/2 \).

We are especially interested in the additive noise case \( (b = 1) \) when \( a \) additionally satisfies non-dissipative condition:

\[
(a(x) - a(y), x - y) \leq -L|x - y|^2, \quad \text{for a non-negative constant } L.
\]

In this case it is known [1] that for any \( H \) exists the initial condition \( x^H_0 \in \mathbb{R}^d \), such that the solution \( X^H_t \) of (1) with \( X^H_0 = x^H_0 \) is stationary.

If a Gaussian field \( B = \{ B^H_t, H \in (0, 1), t \in \mathbb{R} \} \) is given by a Mandelbrot-van Ness representation we establish a pathwise continuous dependence for solutions, with the same initial values, on \( H \in (0, 1) \) and the similar result for the stationary solutions (in this case initial values are not necessary the same). We also discuss some applications to the dynamical systems. This is joint work with M. J. Garrido-Atienza, P. Kloeden and A. Neuenkirch.

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STOCHASTIC APPROXIMATION PROCEDURE WITH INDEPENDENT INCREASE IN LOCAL BALANCE CONDITION

O. I. Shvets¹, Ya. M. Chabanyuk²

The stochastic approximation procedure with impulse perturbation in an ergodic Markov environment in diffusion approximation schema is defined by the stochastic differential equation [1]:

\[
du(t) = a(t)[C(u^x(t); x(t)\varepsilon)]dt + du^\varepsilon(t), u(0) = u_0,
\]

where \( C(u; x), u \in \mathbb{R}^d \) is a regression function, \( u^\varepsilon(t), t \geq 0 \) is a random evolution, and \( \varepsilon \) is a small series parameter.

The impulse perturbation process \( \eta(t) := \varepsilon\eta(t), t \geq 0 \) is defined by generators:

\[
\Gamma(x)\varphi(w) = \int_{\mathbb{R}^d} [\varphi(u + w) - \varphi(u)]\Gamma(u; dw; x), u \in \mathbb{R}^d, x \in X.
\]

Hence, the process \( \eta(t), t \geq 0 \) on the test-functions \( \varphi(u) \in C^0(\mathbb{R}^d) \) is defined by generator:

\[
\Gamma^\varepsilon(x)\varphi(u) = \varepsilon^{-2} \int_{\mathbb{R}^d} [\varphi(u + \varepsilon v) - \varphi(u)]\Gamma(u; dv; x).
\]
Theorem 1. Let there exists the Lyapunov-function $V(u) \in C^3(\mathbb{R}^d)$, for the averaged dynamic system $du(t) = C(u(t))dt$, which satisfy the following conditions:

1. \( C(u)V'(u) < -cV(u), c > 0 \),
2. \( |B(x)V(u)| \leq c_1(1 + V(u)), c_1 > 0 \),
3. \( |\delta^2_{\xi}(x;u)V(u)| \leq c_2(1 + V(u)), c_2 > 0 \),
4. \( |\mathcal{C}(x)R_0\mathcal{C}(x)V(u)| \leq c_3(1 + V(u)), c_3 > 0 \),
5. \( |B(x)R_0\mathcal{C}(x)V(u)| \leq c_4(1 + V(u)), c_4 > 0 \).

where \( \mathcal{C}(x)V(u) = |\mathcal{C}(x) - L|^2 V, L V(u) = \Pi C(x)V'(u), B(x)V(u) = B(x;u)V''(u) \), and \( \|\delta_x^2(x;u)V(u)\| \to 0 \), while \( \varepsilon \to 0 \). Let the local balance condition holds: \( b(u;x) = \int_{\mathbb{R}^d} \mathcal{C}(u;v)dv \equiv 0 \), and the normalization function \( a(t) > 0 \) satisfy the conditions \( \int_0^\infty a(t)dt = \infty, \int_0^\infty a^2(t)dt < \infty \). Then the solution of the stochastic differential equation converges with the probability 1 to the equilibrium point \( u^* \), which is defined by the equation \( C(u^*) = 0 \).

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ON THE INCREASE RATE OF RANDOM FIELDS ON UNBOUNDED DOMAINS

A. I. Slyvka-Tylyshchak

The estimates for distribution of supremum for the normalized \( \varphi \)-sub-Gaussian random fields defined on the unbounded domain are found. In particular, we have obtained the estimates for distribution of supremum for the normalized solution of the hyperbolic equation of mathematical physics.

Theorem 1. Let \( \{\xi(x,t), (x,t) \in V\}, V = [-A;A] \times [0,\infty) \) be a separable random field belonging to \( \text{Sub}_\varphi(\Omega) \). Assume also that the following conditions are satisfied:

1. \( [b_k, b_{k+1}], k = 0,1,\ldots \) is a family of such segments, that
   \[
   0 \leq b_k < b_{k+1} < +\infty, \quad k = 0,1,\ldots \quad V_k = [-A;A] \times [b_k, b_{k+1}], \quad \bigcup_k V_k = V.
   \]
2. There exist the increasing functions \( \sigma_k(h), 0 < h < b_{k+1} - b_k, \) such that \( \sigma_k(h) \to 0 \),
   \[
   \sup_{|x-x_1| \leq h, |t-t_1| \leq h, (x,t),(x_1,t_1) \in V_k} \tau_\varphi(\xi(x,t) - \xi(x_1,t_1)) \leq \sigma_k(h)
   \]
   and \( \int_0^\infty \left( \ln \frac{1}{\sigma_k^{-1}(\varepsilon)} \right) \varepsilon < \infty \), where \( \sigma_k^{-1}(\varepsilon) \) is an inverse function to \( \sigma_k(\varepsilon) \).
3. \( c = \{c(t), t \in R\} \) is some continuous function, such that \( c(t) > 0, t \in R, c_k = \min_{t \in [b_k, b_{k+1}]} c(t) \),
4. \( \sup_k \frac{c_k}{c_k} < \infty, \sup_k \frac{1}{c_k} < \infty \).
5. The series \( \sum_{k=0}^\infty \exp \left\{ -\varphi^*(\frac{sc_k(1-\theta)}{2c_k}) \right\} \) converges for some \( s \) in such a way that \( \sup_k \frac{4sc_k}{sc_k(1-\theta)} < s < \frac{\Delta}{2} \), where \( \varepsilon_k = \sup_{(x,t) \in V_k} \tau_\varphi(\xi(x,t)), k = 0,1,\ldots \)

Then
   \[
P \left( \sup_{(x,t) \in V} \frac{|\xi(x,t)|}{c(t)} > u \right) \leq 2 \exp \left\{ -\varphi^* \left( \frac{c(t)}{u} \right) \right\} \sum_{k=0}^\infty \exp \left\{ -\varphi^* \left( \frac{sc_k(1-\theta)}{2c_k} \right) \right\} = 2A(u),
   \]
   for \( u > \sup_k \frac{L_d(\varphi^*)}{c_k} \cdot \frac{4}{\pi(1-\theta)} \), where \( 0 < \theta < 1 \), \( I_\varphi(\delta) = \int_0^\delta \Psi \left( \left( \frac{\Delta}{\sigma_k^{-1}(\varepsilon)} \right) + 1 \right) + \left( \frac{b_{k+1} - b_k}{2\sigma_k^{-1}(\varepsilon)} + 1 \right) \varepsilon \),

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LARGE DEVIATION PRINCIPLE FOR SDE’S WITH DISCONTINUOUS COEFFICIENTS

D. D. Sobolieva

The talk is devoted to large deviation principle for the solutions to one-dimensional SDE’s. We will discuss some problems that arise in the case of discontinuous coefficients of SDE with some specific forms of discontinuity. It will be shown that in some cases a local time appears in the expression of solution to SDE which influence on the form of the rate function.

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OPTIMAL CONTROL IN THE NONLINEAR STOCHASTIC FUNCTIONAL DIFFERENTIAL ITO-SKOROKHOD EQUATIONS WITH MARKOV PARAMETERS AND EXTERNAL MARKOV SWITCHINGS

V. K. Yasynskyy¹, I. V. Yurchenko²

The stochastic process \( x(t) \equiv x(t, \omega) \in \mathbb{D} \) is defined on the probabilistic basis \( (\Omega, F, F_t, \mathbb{P}) \) as the solution of the following nonlinear stochastic functional differential equation

\[
dx(t) = a(t, x_t, \xi(t), u)dt + b(t, x_t, \xi(t), u)dw(t) + \int_Z c(t, x_t, \xi(t), u)\tilde{\nu}(dz, dt),
\]

with the external Markov switchings \( \{\eta_k, k \geq 0\} \), \( \Delta x(t)_{t=t_0} = g(t_k, x(t_k^-), \xi(t_k^-), \eta(t_k^-)) \), and initial conditions

\[
x(t_0) = \varphi(0, \omega) \in \mathbb{D}, \omega \in \Omega; \quad \xi(t_0) = y(\omega) \in \mathbb{R}^m, \quad \eta(t_0) = h \in \mathbb{H}.
\]

Here \( x_t \equiv \{x(t+\theta, \omega), -h \leq \theta \leq 0; h > 0\} \) is the segment of the strong solution of the equation (1)–(2) with the switchings \( t_k \in S \equiv \{t_k^+, k \in \mathbb{N}\}, \lim_{k \to \infty} t_k = \infty; \mathbb{D} \equiv \mathbb{D}([-\tau, 0], \mathbb{R}^n) \) is the Skorokhod space; \( u \equiv u(t, x, y, h) \in \mathbb{R}^r \) is \( r \)-dimensioned control; \( \xi(t) \equiv \xi(t, \omega) : [t_0, \infty) \times \Omega \to \mathbb{R}^l \) is the Markov process; \( \eta(t) \equiv \eta(t, \omega) : [t_0, \infty) \times \Omega \to \mathbb{R}^m \) is the Markov chain. The sufficient conditions of the stabilization of the (1)–(2) problem’s solution are obtained.

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We prove that there exists a diffusion process whose invariant measure is the fractional polymer or Edwards measure $\nu_{g,H}$, for $dH < 1$. Expressed in terms of the fractional Gaussian measure $\mu_H$ one can write

$$\nu_{g,H} = \frac{1}{Z} \exp \left( -g \int_0^T \int_0^T \delta \left( B^H(s) - B^H(t) \right) \, ds \, dt \right) \mu_H(\omega),$$

with

$$Z = \mathbb{E}_{\mu_H} \left( \exp \left( -g \int_0^T \int_0^T \delta \left( B^H(s) - B^H(t) \right) \, ds \, dt \right) \right),$$

where $B^H$ denotes a $d$-dimensional fractional Brownian motion with Hurst parameter $H < \frac{1}{2}$.

The diffusion is constructed in the framework of Dirichlet forms on infinite dimensional state spaces. Furthermore explicit results of the Wiener-Itô-Segal chaos decomposition of the self-intersection local time of fractional Brownian motion are used.

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STATISTICAL DESCRIPTION FOR SPATIAL STOCHASTIC DYNAMICS: AN OVERVIEW

Dmitri Finkelshtein

We will present an overview of recent results [1–5] in the study of spatial stochastic dynamics: Markov evolutions of locally finite subsets (configurations) of (Euclidean) space. The dynamics were studied using their statistical description as evolutions of probability distributions of points. We will discuss birth-and-death and jump processes on configurations and describe how to derive and solve the evolution equations for the corresponding dynamics of the factorial moments (correlation functions) of the points’ distributions. The general results about existence of evolutions on finite and infinite time intervals will be illustrated by particular examples arising in different applications, in particular, in population dynamics of theoretical ecology. The scheme for the derivation of the so-called kinetic (or, mesoscopic) equations which approximatively describes the exact behaviour of system’s density will be also considered; and the properties of their solutions will be described.

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HYPOCOERCIVITY FOR DEGENERATE KOLMOGOROV EQUATIONS AND APPLICATIONS TO THE LANGEVIN DYNAMICS

M. Grothaus

The aim of this talk is to apply an extended version of the modern, powerful and simple abstract Hilbert space strategy for proving hypocoercivity that has been developed originally by Dolbeault, Mouhot and Schmeiser. It is well-known that hypocoercivity methods imply an exponential decay to equilibrium with explicit computable rate of convergence for degenerate evolution equations. In the stated extension we introduced important domain issues that have not been considered before. Necessary conditions for proving hypocoercivity need then only to be verified on a fixed operator core of the evolution operator. Additionally, the setting is suitably reformulated to incorporate also strongly continuous semigroups solving the Kolmogorov equation as an abstract Cauchy problem. In this way it can be applied to the Langevin dynamics arising in Statistical Mechanics and Mathematical Physics. In this application, the strongly continuous contraction semigroup can be constructed via using Kato perturbation tools. Moreover, via using techniques from the theory of generalized Dirichlet forms, it admits a natural representation as the transition kernel of a diffusion process solving the underlying equation in the martingale sense. Summarizing, we provide the first complete elaboration of the Hilbert space hypocoercivity theorem for the degenerate Langevin dynamics in this hypocoercivity setting.

The results are based on joint work with Patrik Stilgenbauer.

WHICH ACTIVATION FUNCTION OF COOPERATION DESCRIBES HUMAN BEHAVIOR

A. Jarynowski

Properties of cooperation’s probability function in Prisoner’s Dilemma (Game Theory) have impact on evolution of game. Basic model defines that probability of cooperation depends linearly, both on the player’s altruism and the co-player’s reputation. I propose modification of activation function to smooth one (hyperbolic tangent with scaling parameter $a$, which corresponds to its shape) and observe three phases for different range of $a$. (1) For small $a$, strategies seem to randomly change in time and situation of mixed choices (one cooperate and second defect) dominate. (2) For medium $a$, players choose only one strategy for given period of time (the common state can switch to opposite one with some probability). (3) For large $a$, mixed strategy (once defect, once cooperate) is coexisting with common strategies and no change is allowed. I believe that proposed function characterizes better socio-economical phenomena and especially phase 1 and 2 contain most of human behavior.
MULTIVARIATE STATISTICAL EXPERIMENTS WITH PERSISTENT REGRESSION OF INCREMENTS

D. Koroliouk

Multivariate statistical experiments (MSE) with finite number \( M \) of attributes are defined by linear regression function of increments of fluctuations 
\[
\hat{P}_m(k) = P_m(k) - \rho_m, \quad k \geq 0, \quad 0 \leq m \leq M
\]
with respect to equilibrium 
\[
\rho = (\rho_m), \quad 0 \leq m \leq M
\]
by mean of the following relation ([1]):
\[
C^{(m)}_0(\hat{P}(k)) = \sum_{l=0}^{M} [V_l \hat{P}_l(k) - V_m \hat{P}_m(k)], \quad k \geq 0, \quad 0 \leq m \leq M.
\]

The positive values of the directing parameters 
\[
V = (V_m), \quad 0 \leq m \leq M
\]
provide the existence of the equilibrium state (Theorem 1) and the normal approximation of the fluctuation increments (Theorem 2) by a number of observations \( N \to \infty \).

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EVOLVING STOCHASTIC PROCESSES AND THEIR APPLICATIONS

Maria Jo˜ ao Oliveira

We consider the non-equilibrium dynamics for the Widom-Rowlinson model in the continuum and construct the time evolution in terms of generating functionals. This is carried out by an Ovsjannikov-type result in a scale of Banach spaces, which leads to a local (in time) solution. The Lebowitz-Penrose-type scaling of the dynamics is analyzed and the system of the corresponding kinetic equations is derived. This talk is based on the recent joint work [1].

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A STOCHASTIC MODEL OF MULTI-BUFFER STORAGE SYSTEM WITH SEMI-MARKOV INPUT RATE

M. Ya. Postan

A storage system consisting of \( n \) consequently connected buffers is under consideration. It is assumed that the capacities of all buffers are infinite. At the first buffer of storage system the homogeneous product comes from outside with the rate depending on the state of semi-Markov process \( Y(t) \) with the finite set of states \( S \). That is, this rate equals to \( W_{1i} \) at moment \( t \) if \( Y(t) = i \in S \). From the \( k \)th buffer to the \( (k+1) \)th one product comes with the rate \( W_k, W_k > W_{k+1} > 0, k = 2, 3, ..., n-1 \). The \( n \)th buffer product leaves the system with the rate \( W_n \). We assume also that
\[
S^+ = \{ i : W_{1i} > W_2,i \in S \} \neq \emptyset.
\]
The similar system was studied in [1] for the case when input flow of product is described by the Levy process. Let \( \xi_k(t) \) be the storage level of product in the \( k \)th buffer at moment \( t \). The ergodicity conditions and stationary distribution of the random vector \( (\xi_1(t), ..., \xi_n(t)) \) are studied.
INFINITESIMAL GENERATOR FOR THE DISCRETE HIDDEN MARKOV MODEL

S. A. Semenyuk¹, Ya. M. Chabanyuk², U. T. Khimka³

Consider hidden Markov model (HMM) as a two-component process \( z_t = (x_t, y_t) \). Here the HMM state process \( x(t), t \geq 0 \) is uniformly ergodic Markov process in the standard phase space \( (X, X) \) with a generator [1]:

\[
Q_\varphi(x) = q(x) \int_X P(x, dy) \left[ \varphi(y) - \varphi(x) \right], \quad \varphi \in B(X),
\]

\( B \) denotes Banach space of real bounded functions with supremum norm \( \| \varphi \| = \max_{x \in X} |\varphi(x)| \). Let process \( x(t) \) have stationary distribution \( \pi(C), C \in X \).

And the HMM observation process \( y(t), t \geq 0 \) is a sequence of random variables with set of probability mass functions:

\[
b_x(y) = P[y_t = y|x_t = x], y \in Y.
\]

It can be shown that the process \( z_t \) is Markovian, moreover, it is ergodic and stationary if \( x_t \) is ergodic and stationary [2].

Lemma 1. Infinitesimal generator for the two-component Markov process \( z_t = (x_t, y_t), t \geq 0 \) has the form:

\[
L_\varphi(x, y) = q(x) \int_X P(x, ds) \sum_{p \in Y} \left[ b_s(p) \varphi(s, p) - b_x(p) \varphi(x, p) \right], \quad \varphi \in B(X,Y).
\]

Obtained result allows to consider and study complex stochastic systems that depends on hidden Markov models [3].

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STOCHASTIC OPTIMIZATION

LINEAR UNCONDITIONAL PROBLEM OF COMBINATORIAL STOCHASTIC OPTIMIZATION ON ARRANGEMENTS: CONSTRUCTION AND SOLVING

T. M. Barbolina

We propose a new approach to construction of optimization problems under probability uncertainty which is based on linear order on a set of discrete random variable. This approach gives more possibilities when optimality criterion is forming. Under such approach author formulates the stochastic optimization problem with linear objective function on combinatorial set of arrangements when unknowns are discrete random variables. Properties of this problem are investigated. For special cases of such problems we propose ways of solution’s finding without exhaustive search in polynomial time.

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IDENTIFICATION FOR NONLINEAR STOCHASTIC SYSTEMS

O. M. Deriyeva¹, S. P. Shpyga²

Consider the stochastic system
\[ dX(s, t) = \theta(s, t)X(s, t)ds + dB^H(s, t) \]
where \( X(0, 0) \) is a Gaussian random variable, \( B^H(s, t) \) is a fractional Brownian field with parameter \( H = (\alpha, \beta) \), \( \alpha, \beta \in (1/2, 1) \). Suppose that the process \( X \) is observable on \([0, \tau]\), and \( X_i, 1 \leq i \leq n \) is a random sample of it’s \( n \) independent observations. By the method of sieves the restricted maximum likelihood estimation \( \hat{\theta}^{(n)}(.) \) of \( \theta(.) \) can be found. This estimation is consistent and asymptotically normal.

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FUTURES HEDGING

V. M. Gorbachuk

Theorem 1. If a person wants to buy a security, then she can: 1) buy it for immediate delivery in the period 1 at the spot price \( S_1 \); 2) place an order for later delivery in the period \( t \), or buy at the future price \( F_t \). In the case 2) a person gets the same security as in the case 1), but: a) does not pay for this security immediately; b) does not receive any dividend and interest payments \( D_t \), this security provides in the periods 1, 2, ..., \( t \). This implies the following relationship between the spot price \( S_1 \), the future price \( F_t \), the dividend and interest payments \( D_t \): \( F_t/(1 + r_f)^t = S_1 - PV(D_t) \), where \( r_f \) is the risk-free interest rate in the periods 1, 2, ..., \( t \). PV(.) stands for present (discounted) value. This relationship is correct if the (futures) contract is marked to market; if the contract is marked to market, then it depends on the interest rates in the periods 1, 2, ..., \( t \), which is not taken into account in practice.

Theorem 2. The difference between buying commodities today and buying commodity futures is more complicated because: i) the payment is delayed, and therefore a buyer can receive the interest on her money; ii) there is no need to store commodities, and therefore a buyer of futures contract saves her costs \( SC_1 \) for warehouse, wastage etc. (storage costs). On other hand, the financial futures contract does not give the convenience yield \( CY_1 \), or the value of ability to real actions. It does not make sense to hold the financial futures contract at a higher price of futures, and it does not make sense to hold the commodity at a lower price of futures: \( F_t/(1 + r_f)^t = S_1 + PV(SC_1) - PV(CY_1) \). Small inventories imply the low value of \( SC_1 \). The net convenience yield \( PV(CY_1) - PV(SC_1) \) plays the role of \( PV(D_t) \).

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The author is supported by the National Academy of Sciences of Ukraine, the project Integrated food, energy, water security management for sustainable social, economic and environmental developments.
The mathematical apparatus of polyhedral coherent risk measures (PCRM) introduced for discrete random variables [1] is proposed for the risk assessment. The PCRM class has a number of advantages. It is a subset of the class of coherent risk measures having theoretically attractive properties. Besides, its use allows us to reduce portfolio optimization problems to linear programming problems (LPP) [1].

The PCRM class contains a number of known risk measures. One of the most famous such measures is Conditional Value-at-Risk, which in recent years is seen as a successful replacement for Value-at-Risk. The last one has been widely used in finance and insurance for a long time, but currently it is seriously criticized for a number of shortcomings.

The mathematical tool of PCRM allows constructing such risk measures not only in the conditions of known distributions of random variables, but under incomplete information on the distributions [2]. For example, for case of imprecise scenario probabilities. In the latter case it is made for expected utility [4] is discussed. We show how these problems are reduced to LPP not only in the conditions of uncertainty using such measures can be reduced to appropriate LPP.

The examples of constructing PCRM are considered. Their use allows us to unite problems of stochastic programming and robust optimization within the overall approach. It is shown how linear optimization problems under uncertainty using such measures can be reduced to appropriate LPP.

The application of this apparatus for portfolio optimization problems on risk-reward ratio [3] and maximizing expected utility [4] is discussed. We show how these problems are reduced to LPP not only in the conditions of known distributions of random variables, but under imprecise scenario probabilities. In the latter case it is made for the corresponding robust optimization problems. Reducing the initial decision-making problems to LPP allows us efficiently to solve them even under large dimensions.

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CONVERGES OF OPTIMIZATION PROCEDURE WITH IMPULSIVE PERTURBATIONS

V. R. Kukurba1, Ya. M. Chabanyuk2, I. S. Budz3

Continuous stochastic optimization procedure (SOP) with semi-Markov’s switchings and impulsive perturbations is defined by the evolution equation [1]:

$$du^\varepsilon(t) = a(t)[\nabla_{\kappa(t)} C(u^\varepsilon(t); x(t/\varepsilon^3))dt + \varepsilon \eta^\varepsilon(t)],$$  \hspace{1cm} (1)

where $\nabla_{\kappa(t)} C(u; x) = \left(\frac{C(u(x(t), x(t)) - C(u(x(t))x))}{2\varepsilon^3}\right)$, $u \in R$. The regression function $C(u; x)$ depends on uniformly ergodic semi-Markov process $x(t) > 0$, $t \geq 0$, in the dimensional space phase of states $(X, X)$, $u^\varepsilon(t), t \geq 0$ is a random evolution, $\varepsilon$ is a scheme parameter. Impulsive perturbation process $\eta^\varepsilon(t), t \geq 0$ and its generator are defined in [1].

Consider conditions of existing Lyapunov function $V(u) \in C^4(R)$, that satisfies exponential stability of average system

$$\frac{du(t)}{dt} = C(u(t)), C(u) := \int_X \pi(dx)C(u; x).$$  \hspace{1cm} (2)

Also we defined additional conditions for regression function and use Kramer condition for distribution function to determine conditions of weak converges of SOP (1) for extremum point of average system (2)

$$P\{ \lim_{t \to \infty} u^\varepsilon(t) = 0 \} = 1.$$

References

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MINIMAX FILTERING PROBLEM FOR RANDOM PROCESSES WITH STATIONARY INCREMENTS

M. M. Luz¹, M. P. Moklyachuk²

We consider a random process \( \xi(t) \), \( t \in R \), with stationary nth increments \( \xi^{(n)}(t, \tau) \), \( t \in R, \tau > 0 \), and a stationary random process \( \eta(t) \), \( t \in R \), uncorrelated with \( \xi(t) \). The problem of optimal mean-square filtering of a linear functional \( A\xi = \int_0^\infty a(t)e^{-\lambda t}dt \) that depends on the unknown values of the process \( \xi(t) \) from observations of the process \( \xi(t) + \eta(t) \) at points \( t \leq 0 \) is discussed.

Spectral characteristic of the optimal estimate \( \hat{A}\xi \) is calculated by the formula

\[
h_s(\lambda) = \frac{A(\lambda + i\lambda)^n - (-i\lambda)^n g(\lambda)}{(1 + \lambda^2)^n f(\lambda) + \lambda^{2n} g(\lambda)} \int_0^\infty \left( (\sum_n^\infty \lambda^{-n}) (\sum_n^\infty \lambda^2)^n \right) \frac{e^{i\lambda t} dt}{(1 - e^{i\lambda^2} n) f(\lambda) + \lambda^{2n} g(\lambda)},
\]

where \( A(\lambda) = \int_0^\infty a(t)e^{-\lambda t}dt \), linear operators \( S_n^\infty \in L_2([-\tau; \infty)) \), \( P_n^\infty \in L_2([0; \infty)) \) are determined by the spectral densities \( f(\lambda) \) and \( g(\lambda) \) of the processes \( \xi(t) \) and \( \eta(t) \).

In the case where spectral densities \( f(\lambda) \) and \( g(\lambda) \) are not known, but a set of admissible spectral densities is given, we propose relations that determine the least favorable spectral densities and the minimax spectral characteristic for optimal lineal estimating of the functional \( A\xi \).

Filtering problem for stochastic sequences with stationary increments is considered in [1], [2]. Random processes with stationary increments are considered in [3].

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LLN FOR RANDOM MAPPINGS AND ITS APPLICATION IN STOCHASTIC OPTIMIZATION

V. I. Norkin

In the report we develop a calculus for graphically convergent sequences of semi-continuous mappings. Graphical convergence of mappings is a generalization of uniform convergence and means set convergence of their graphs to the graph of the limit mapping. As a motivation we show that graphical convergence is the basic ingredient for validating approximation schemes for solution of generalized equations/inclusions and optimization problems with equilibrium constraints. Sequences of mappings appear here as approximations of feasible sets, epi-graphs, subgradients and normal cone mappings. As a basic area of applications we consider stochastic optimization and related stochastic variational analysis problems involving expectations of random mappings. Graphically convergent sequences of mappings arise here in the framework of the so-call sample average approximation scheme and as a result of application of the law of large numbers to random mappings. So we review some classical and recent results on the law of large numbers for random sets and mappings, see details in [1, 2]. A calculus is need when establishing graphical convergence of composite mappings such as point-wise sums, products, unions, intersections and compositions. To obtain graphical convergence of a composition we utilize both graphical and point-wise convergences of the ingredient mappings. For stochastic mappings we establish conditions of graphical convergence with probability one.

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NUMERICAL TECHNIQUES FOR ESTIMATION OF UNKNOWN GIBBS DISTRIBUTION PARAMETERS

Oleksandr Samosyonok

Consider a model of Markov random field with local interaction given by \((S, N, H)\), where \(S\) is a set of vertices, \(N\) is a neighborhood system and \(H\) is a set of Gibbs potentials. Let \(X = (X_1, ..., X_{|S|})\) be multidimensional random variable, where every component \(X_i\) is one-dimensional random variable taking values \(x_i \in A\) from the finite set \(A\). If for every element \(s\) of the field we can sign his neighbors \(\partial s\) then consider set \(\partial = \{\partial s : s \in S\}\) to be a neighboring system. Subset of elements of random fields corresponding to full connected subgraph of neighboring graph is called
clique $\chi$: $\forall s, t \in \chi, s \neq t \Rightarrow t \in \partial s$. Number of elements in this subset is clique order. Also, it is natural to assume that the clique set can be identified by the geometric arrangement of the interacting elements. Denote set of cliques $\mathcal{C} = \{C_i\}$ and assume that this set is finite. Under all mentioned considerations probability of some system state $x$ could be represented as Gibbs distribution:

$$P(x) = Z(v)^{-1} \exp \left( - \sum_{j=1}^{[\mathcal{C}]} \sum_{\chi \subset S} H_{\chi}(x_{\chi}, v^j) \right), \quad v = (v^j)_{j=1}^t,$$

where $x_{\chi}$ is a configuration of clique $\chi$, so $x_{\chi} = \{x_{i_1}, x_{i_2}, ..., x_{i_{|\chi|}}\}$, $i_j \in \chi$, $Z(v)$ is a normalizing factor.

Consider problem of estimation unknown parameter $v$ according to given observations. Implementing of traditional estimation methods based on derivative computation for Gibbs distribution is quite complex problem since its normalizing factor $Z$ depends on each state of every element of the field. To simplify this problem it is proposed to combine well-known stochastic quasi-gradient methods with stochastic simulation algorithms. Consider the logarithmic function of negative maximum likelihood function taking into account Gibbs distribution (1):

$$L_n(x, v) = \sum_{i=1}^{n} \left( \sum_{\chi \subset S} \sum_{j=1}^{[\mathcal{C}]} H_{\chi}(x_{\chi i}, v^j) \right) + n \ln Z(v),$$

and consider problem of finding its minimal value $v_n = \arg \min L_n(x, v)$. If $v_n$ is consistent estimator of minimum of function $L_n(x, v)$, then stochastic quasi-gradient could be written in next form:

$$\hat{\nabla} L_n(x^i, v) = \left( \sum_{\chi \subset S} \frac{\partial H_{\chi}(x^i_{\chi}, v^j)}{\partial v^j} - E \left( \sum_{\chi \subset S} \frac{\partial H_{\chi}(x_{\chi}, v^j)}{\partial v^j} \right) \right)_{j=1}^t.$$

As a result the most complicated part of algorithm is mathematical mean calculation. For mathematical mean calculation any method of stochastic simulation could be used. Usage of wide-spread Monte-Carlo method is followed by some negative consequences. At first, it is absence of information about minimal size of data sample enough for required accuracy and, second, complexity of generating random field in case of Gibbs distribution. Both of these problems could be eliminated by implementing so called Markov chain Monte-Carlo (MCMC) algorithms, which, actually, are modified methods of Monte-Carlo. Another very effective method for Gibbs parameter estimation is so-called pseudolikelihood method. This principle at first was implemented by J. Besag. Unlike traditional likelihood method, J. Besag proposed to estimate unknown parameters by the product of conditional probabilities.

**References**


A MULTIDIMENSIONAL COMPARISON THEOREM FOR SDE WITH DRIFT

S. Anulova

A problem of controlling a group of independent identical stochastic agents is considered. Such problems arise in mathematical economics, in robotics (swarms of mini-robots). The aim is to hold the group as high as possible and the optimal policy allocates the control drift so: wholly to the lowest agent. This solution is found by the comparison method for solutions of SDE using the technique of the Skorokhod reflection operators. Comparison with partial ordering (in $\mathbb{R}^d$) is a pioneering approach.

This control model is a representative of processes with rank-based characteristics. They are intensively explored at the current time, see articles by I. Karatzas, T. Ichiba, B. Jourdain, J. Reygner, S. Pal, J. Pitman et al.

The results of the preceding author’s articles in Proceedings of conferences CDC 2010 and ECC 2014 are generalized: the dynamics of the agent is extended to state dependent (non-decreasing drift at the current time, see articles by I. Karatzas, T. Ichiba, B. Jourdain, J. Reygner, S. Pal, J. Pitman et al.

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similar control criteria are also considered and analogous optimal strategies are found under corresponding restrictions on the drift of the agent.

The talk is based on reference [1].

Ehsan Azmoodeh

At the moment, normal approximation using Stein’s method and Malliavin calculus for a sequence of multiple integrals of a fixed order is well understood, see for example the recent monograph [2]. Consider the random variables $F_\infty$ of the type

$$F_\infty = \sum_{i=1}^{k} \alpha_i (N_i^2 - 1),$$

where $k$ is a finite integer, the $\alpha_i$, $i = 1, ..., k$, are pairwise distinct real numbers, and $\{N_i : i = 1, ..., k\}$ is a collection of i.i.d. $\mathcal{N}(0, 1)$ random variables. We recall that such random variables are typical elements of the second Wiener chaos. The main objective of the talk is to address the following central question: is it possible to prove convergence in distribution towards the target distribution $F_\infty$ in terms of convergence of finitely many moments/cumulants for the sequence $\{F_n\}_{n \geq 1} = \{I_p(f_n)\}_{n \geq 1}$ of multiple integrals of the fixed order $p \geq 2$?

As a breakthrough example, we discuss in more details the case where $k = 2$, $\alpha_1 = -\alpha_2 = \frac{1}{2}$ and $F_\infty \overset{\text{law}}{=} N_1 \times N_2$. The talk is based on reference [1].

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ON THE REDUCTION IN OPTIMAL STOPPING WITH INCOMPLETE DATA

P. Babilua1, B. Dochviri2, V. Jaoshvili3

Let us consider the optimal stopping problem for partially observable conditional Gaussian process $(\theta, \xi) = (\theta, \xi_t)$, $t \geq 0$.

$$d\theta_t = a(t, \xi)\theta dt + b(t, \xi) d\omega_1(t),$$

$$d\xi_t = A(t, \xi)\theta dt + c d\omega_2(t),$$

where $\epsilon > 0$, $\omega_1$ and $\omega_2$ are independent Wiener processes [1]. It is assumed that $\theta$ is the nonobservable process, $\xi$ is the observable process. Let the gain function $g(x) = x$, $x \in R$, and introduce the payoffs: $S^0 = \sup_{\tau} E\theta_\tau$, $\tau \in \mathbb{R}^0$; $S^\epsilon = \sup_{\tau} E\theta_\tau$, $\tau \in \mathbb{R}^\epsilon$, where $\mathbb{R}^0$, $\mathbb{R}^\epsilon$ are the classes of stopping times with respect to $\sigma$-algebras $\mathcal{F}_t^\epsilon = \sigma(\theta_s, s \leq t), \epsilon \geq 1$. 

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Let us consider homogeneous branching process with continuous time and migration [1]. At any time $t$ either with probability $P_k(t)$, $k$ particles migrate into population, or with probability $Q_r(t)$, $r$ particles emigrate from population ($r = 1, m$) $\sum_{k=0}^{\infty} P_k(t) + \sum_{r=1}^{m} Q_r(t) = 1$. Let us denote the process of term's migration and emigration as $\zeta(t)$, in other words $P(\zeta(t) = k) = P_k(t)$, $P(\zeta(t) = -r) = Q_r(t)$, $r = 1, \ldots, m$.

Furthermore, choose $\Delta t$ ($\Delta t \to 0$). Suppose, that $P_0(\Delta t) = 1 + p_0 \Delta t + o(\Delta t)$, $P_k(\Delta t) = p_k \Delta t + o(\Delta t)$, $k \geq 1$, $Q_1(\Delta t) = q_1 \Delta t + o(\Delta t)$, $Q_r(\Delta t) = q_r \Delta t + o(\Delta t)$, $r = 2, \ldots, m$.

Denote by $\xi(\Delta t)$ ($i = 1, \mu(t)$) the number of offspring of $i$-particle at the point of time $\Delta t$. Then

\[ P(\xi(\Delta t) = n|\xi(0) = 1) = H_n(\Delta t) = \begin{cases} n h_n \Delta t + o(\Delta t), & n = 0, 2, \ldots, \\ 1 + n h_n \Delta t + o(\Delta t), & n = 1, \end{cases} \]

where $\sum_{n=0}^{\infty} h_n = 0, h_1 \leq 0, h_j \geq 0$ ($j = 0, 2, \ldots$), $\sum_{n=0}^{\infty} H_n(\Delta t) = 1$ [2].

Denote $F(t, s) = MS^{\xi(t)} = \sum_{n=0}^{\infty} H_n(t) s^n$, $|s| \leq 1$, $h(s) = \sum_{n=0}^{\infty} h_n s^n$, $|s| \leq 1$.

Let $\tau^*$ be the first moment out of the zero, $A(t) = F^{\infty}(t, s) = \sum_{n=1}^{\infty} \sum_{j=1}^{n} P_j(t)^n$.

\[ F_{\mu}(t + \Delta t, s|\tau^* = t) = A(t + \Delta t)(A'(t)h(s) - A(t)p_0 + B(t)) + o(\Delta t). \]

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ON THE METHOD OF STOCHASTIC IDENTIFICATION OF ALMOST PERIODIC SIGNAL

G. D. Bila

We consider the problem of stochastic identification of almost periodic function when it is known that useful signal is transmitted through the communication channel and is subjected to the random noise. We consider the following structure of the model of output signal

\[ x(t) = a_0(t) + G(n(t)), \quad 0 \leq t \leq T, \]

where the useful signal $a_0(t) \in K$, $K$ is a set of real almost periodic functions, and random noise $G(n(t))$, $t \in R$, given by a functional of a homogeneous gaussian random process $n(t)$, $t \in R$, with a long-range dependence. For
HEAT KERNEL ESTIMATES FOR UNIMODAL LÉVY PROCESSES

Krzysztof Bogdan

I will report joint results with Tomasz Grzywny and Michał Ryznar published last year in a series of three papers. We estimate the heat kernel $p_D$ of smooth open sets $D \subset \mathbb{R}^d$ for the radially unimodal pure-jump Lévy processes $\{X_t, t \geq 0\}$ with infinite Lévy measure and weakly scaling Lévy-Khintchine exponent $\psi$. The estimates have a form of explicit factorization involving the transition density $p$ of the Lévy process (on the whole of $\mathbb{R}^d$), and the survival probability $\mathbb{P}^x(\tau_D > t)$, where $\tau_D = \inf\{t > 0 : X_t \notin D\}$ is the time of the first exit of $X_t$ from $D$. For instance, if $\psi$ has global lower and upper scalings, and $D$ is a $C^2$ halfspace-like open set, then

$$p_D(t, x, y) \approx \mathbb{P}^x(\tau_D > t) \cdot p(t, x, y) \mathbb{P}^y(\tau_D > t), \quad t > 0, x, y \in \mathbb{R}^d,$$

where

$$p_D(t, x, y) \approx \psi^{-1}(t/\lambda d) \wedge \frac{t \psi(1/|x|)}{|x|^d},$$

and $\delta_D(x) = \text{dist}(x, D^c)$. The scaling conditions are understood as follows. We say that $\psi$ satisfies the weak lower scaling condition at infinity (WLSC) if there are numbers $\alpha > 0, \beta \geq 0$, and $\zeta \in (0, 1)$, such that $\psi(\lambda \theta) \geq C \lambda^\alpha \psi(\theta)$ for $\lambda \geq 1, \theta > \theta_0$. We write $\psi \in \text{WLSC}(\alpha, \beta, \zeta)$ or $\psi \in \text{WLSC}$. If $\psi \in \text{WLSC}(\alpha, 0, \zeta)$, then we say that $\psi$ satisfies the global WLSC. The weak upper scaling condition at infinity (WUSC) means that there are numbers $\pi < 2, \theta \geq 0$, and $\zeta \in [1, \infty)$ such that $\psi(\lambda \theta) \leq C \pi \psi(\theta)$ for $\lambda \geq 1, \theta > \theta_0$. In short, $\psi \in \text{WUSC}(\pi, \theta, \zeta)$ or $\psi \in \text{WUSC}$. Global WUSC means $\text{WUSC}(\pi, 0, \zeta)$. Here $\psi$ is a radial function, which defines $\psi(\theta) = \psi(\xi)$ for $\xi \in \mathbb{R}^d, |\xi| = \theta$.

EXCEEDANCE TIMES OF PERPETUITY SEQUENCES

D. Buraczewski

We study the large exceedance probabilities of the perpetuity sequence

$$Y_n := B_1 + A_1 B_2 + \cdots + (A_1 \cdots A_{n-1}) B_n,$$

where $(A_i, B_i) \subset (0, \infty) \times \mathbb{R}$ is a sequence of iid random variables. Estimates for the stationary tail distribution of $\{Y_n\}$ have been developed in the seminal papers of Kesten [3] and Goldie [2]. Specifically, it is well-known that if $M := \sup_n Y_n$, then

$$\mathbb{P}\{M > u\} \sim C_M u^{-\xi}$$

as $u \to \infty$. Recently in [1] we derived sharp asymptotic estimates for the large exceedance times of $\{Y_n\}$. Letting $T_u := (\log u)^{-1} \inf\{n : Y_n > u\}$ denote the normalized first passage time, we study $\mathbb{P}\{T_u \in G\}$ as $u \to \infty$ for sets $G \subset [0, \infty)$.

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ON \(L^p\)-CRITERION FOR TESTING HYPOTHESIS ON COVARIANCE FUNCTION OF A RANDOM SEQUENCE

T. O. Ianevych

Let us consider a stationary (in a weak sense) Gaussian random sequence \(\{\gamma_i, i \geq 1\}\) that for all \(i\) \(\mathbb{E}\gamma_i = 0\) and \(\mathbb{E}\gamma_i\gamma_{i+m} = B(m), m \geq 0\) be its covariance function. As a estimator of the covariance function \(B(m)\) we use \(\hat{B}_N(m) = \frac{1}{N-m} \sum_{i=1}^{N-m} \gamma_i\gamma_{i+m}\). The estimator \(\hat{B}_N(m)\) is unbiased: \(\mathbb{E}\hat{B}_N(m) = \frac{1}{N-m} \sum_{i=1}^{N-m} \mathbb{E}\gamma_i\gamma_{i+m} = B(m)\).

It is appeared that the random variables \(\Delta_N(m) = \hat{B}_N(m) - B(m)\) is square Gaussian. For details on square Gaussian random variables (see [1]).

Suppose, we observe \(N\) consecutive observations \(\gamma_1, ..., \gamma_N\) of some random sequence \(\{\gamma_i, i \geq 1\}\) and have some reasons to believe that this sequence is centered, stationary in a weak sense and Gaussian with covariance function \(B(m)\).

**Criterion 1.** Let the main hypothesis \(H_0\) be that \(B(m), m \geq 0\) is the covariance function of the sequence \(\{\gamma_i, i \geq 1\}\). And the alternative hypothesis \(H_a\) is the opposite statement. If for some significance level \(\alpha\), some fixed \(p > 0\) and \(n < \infty\)

\[
\sum_{m=1}^{n} \left| \frac{\Delta_N(m)}{\text{Var}\Delta_N(m)} \right|^p > \delta_\alpha,
\]

where \(\delta_\alpha\) is a value for which the following equality holds true

\[
2 \sqrt{1 + \frac{2\delta_\alpha}{n^{1/p}}} \exp \left\{ - \frac{\delta_\alpha}{\sqrt{2} n^{1/p}} \right\} = \alpha,
\]

implying that \(\delta_\alpha > \frac{p^{1/p}}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{2}{p}}\right)\), then the \(H_0\) is rejected and not – otherwise.

**REFERENCES**


**FUNCTIONAL LIMIT THEOREMS FOR PERTURBED RANDOM WALKS**

Alexander Iksanov

Let \((\xi_k, \eta_k)_{k \in \mathbb{N}}\) be a sequence of i.i.d. two-dimensional random vectors with arbitrary dependence of the components.

A random sequence \((T_n)_{n \in \mathbb{N}}\) defined by

\[ T_n := \xi_1 + \cdots + \xi_{n-1} + \eta_n, \quad n \in \mathbb{N} \]

is called a perturbed random walk.

I intend to discuss a functional limit theorem for \(Y_n(\cdot) := \max_{0 \leq k \leq n} (\xi_1 + \cdots + \xi_k + \eta_{k+1})\), properly normalized, in the situation when contributions of \(\max_{0 \leq k \leq n} (\xi_1 + \cdots + \xi_k)\) and \(\max_{1 \leq k \leq n+1} \eta_k\) to the asymptotic behavior of \(Y_n\) are comparable.

The other problem to be addressed is weak convergence in the Skorokhod space of divergent perpetuities

\[ Z_n(\cdot) := Q_1 + M_1Q_2 + \cdots + M_1M_2 \cdots M_{n}Q_{[n]+1}, \]

where \((M_k, Q_k)_{k \in \mathbb{N}}\) is a sequence of i.i.d. two-dimensional random vectors with arbitrary dependence of the components.

The presentation is based on two recent papers [1] and [2].

**REFERENCES**


**INVARIANCE AND UNIMODULARITY IN THE THEORY OF RANDOM NETWORKS**

Vadim A. Kaímanovich

The natural state space for measures associated with random infinite graphs is the space \(G_*\) of rooted locally finite graphs. Although there is no group action on \(G_*\), this space is endowed with the natural “root moving” equivalence relation \(\mathcal{R}\), so that one can talk about measures invariant with respect to this equivalence relation. This notion of invariance for measures on \(G_*\) was introduced by the author [4].
By drawing their inspiration from the “mass transfer principle" of Häggström [3], Benjamini and Schramm [2] defined an “intrinsic mass transport principle“ for probability measures on $\mathcal{G}_*$. This notion has now become quite popular among probabilists under the name of “unimodularity" (e.g., see [1]). Although for measures concentrated on the set of rigid graphs (i.e., those with trivial group of automorphisms) the notions of invariance and unimodularity coincide, they do differ in general, as one can easily see by looking just at finite graphs. The purpose of the present research communication is to clarify the relationship between invariance and unimodularity in full generality.

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Ya. M. Khusanbaev

Let for each $n \in \mathbb{N}$ \( \{\zeta_{k,j}^{(n)}, k, j \in \mathbb{N}\} \) and \( \{\tau_k^{(n)}, k \in \mathbb{N}\} \) be two independent families of random variables taking nonnegative integer values. Let \( \{\zeta_{k,j}^{(n)}, k, j \in \mathbb{N}\} \) be independent and identically distributed random variables, \( \{\tau_k^{(n)}, k \in \mathbb{N}\} \) be a stationary process in the wide sense.

Define \( \{Z_k^{(n)}, k \in \mathbb{N}\} \), \( n \in \mathbb{N} \) recursively by the formula

\[ Z_0 = 0, \quad Z_k^{(n)} = \sum_{j=1}^{\zeta_{k,j}^{(n)}} \tau_k^{(n)} + \tau_k^{(n)}, \quad k, n \in \mathbb{N}. \]

The sequence \( \{Z_k^{(n)}, k \in \mathbb{N}\} \) is called the branching process with an immigration. Define the random step function \( Z_n(t), t \geq 0 \) with trajectories in the Skorokhod space \( D(0, \infty) \) as \( Z_n(t) = Z_{[nt]}^{(n)}, t \geq 0, \) where \([a]\) means the integer part of \( a \). Suppose that

\[ m_n = \mathbb{E}\zeta_{1,1}^{(n)}, \quad \sigma_n^2 = \text{var}\zeta_{1,1}^{(n)}, \quad \gamma_n = \mathbb{E}\tau_1^{(n)}, b_n^2 = \text{var}\tau_1^{(n)}, \quad \rho_n(k) = \text{cov}\left(\tau_1^{(n)}, \tau_k^{(n)}\right) \]

are finite for all \( n \in \mathbb{N}. \)

Now we formulate the result.

Theorem 1. Let

A. \( m_n = 1 + \frac{1}{n} + O\left(\frac{1}{n}\right) \) as \( n \to \infty \) for some \( a \in \mathbb{R}; \)

B. \( \sigma_n^2 \to 0 \) as \( n \to \infty; \)

C. \( \gamma_n \to \gamma \geq 0, b_n^2 \to b^2 \geq 0 \) as \( n \to \infty; \)

D. \( \frac{1}{n} \sum_{k=1}^{n} |\rho_n(k)| \to 0 \) as \( n \to \infty. \)

Then the weak convergence \( n^{-1}Z_n \to \mu \) as \( n \to \infty \) holds in the Skorokhod space \( D(0, \infty) \), where the limit process \( \mu \) is defined by the relation \( \mu = \gamma \int_0^t e^{\mu u} du. \)

Ya. M. Chabanyuk

Consider the stochastic system with diffusion perturbation which is defined by stochastic differential equation [1]:

\[ du(t) = C(u^\varepsilon(t); x(t/\varepsilon^3))dt + \varepsilon^{-1}C_0(u^\varepsilon(t); x(t/\varepsilon^3))dt + \varepsilon^{1/2}\sigma(u^\varepsilon(t); x(t/\varepsilon^3))dw(t), \]

A. V. Kinash

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where \( u^*(t), t \geq 0 \) is a random evolution, \( u \in R^d \); \( C_0(u; x) \) is a singular perturbation of the regression function \( C(u; x) \); \( x(t), t \geq 0 \) is a Markov process in the dimensional space phase of states \((X, \mathbf{X})\) with stationary distribution \( \pi(B), B \in \mathbf{X}; \sigma(u; x) \) is a diffusion.

Limited evolution of initial system has the form

\[
du(t) = C(u)dt + \varepsilon B(u)du(t)/2,
\]

where \( C(u) = \int_X C(u; x)\pi(dx). \) Limited diffusion \( \sigma \) is determined from the ratio \( \sigma(u)\sigma^*(u) = B(u) \), where \( B(u) = \int_X \sigma^2(u; x)\pi(dx) \).

**Theorem 1.** Let there Lyapunov function

\[
\int_0^t C(u)\pi(dx)dt,
\]

where \( C(u) = \int_X C(u; x)\pi(dx) \). Limited diffusion \( \sigma \) is determined from the ratio \( \sigma(u)\sigma^*(u) = B(u) \), where \( B(u) = \int_X \sigma^2(u; x)\pi(dx) \).

Under the condition of balance \( \int_X \pi(dx)C_0(u; x) \equiv 0 \) and inequalities \( C(u)V'(u) < c_1V(u), \sup_{u \in R^d} \| \sigma(u) \| < c_2, c_1 > 0, c_2 > 0 \), initial system is asymptotic dissipative [2].

**References**


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**STOCHASTIC MODELS FOR RECORDS**

O. I. Klesov

The talk is devoted to some extensions of classical results for records constructed from a sequence of independent identically distributed random variables. The extensions include the so called \( F^\alpha \) and random \( F^\alpha \) schemes.

The talk is based on the results obtained in [1,2].

**References**


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**THE METHOD OF THE EMPIRICAL MEANS IN THE PROBLEMS OF STOCHASTIC OPTIMIZATION AND ESTIMATION**

P. S. Knopov

In stochastic optimization and identification problems it is not always possible to find the explicit extremum for the expectation of some random function. One of the methods for solving this problem is the method of empirical means, which consists in approximation of the existing cost function by its empiric estimate, for which one can solve the corresponding optimization problem. In the talk we discuss some problems in the method of empirical means related to consistency, asymptotic distribution and large deviations in some stochastic optimization models and estimation theory.

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ON PATHS PROPERTIES OF SOME LEVY-TYPE PROCESSES

V. Knopova

The talk is devoted to the Hausdorff dimension of the level and collision sets of some Levy-type process $X$. To obtain the upper and lower bounds on the Hausdorff dimension of the level set, we use the structure of the transition probability density of $X$, the time change, and the known results on the dimension of the range of the subordinator. For the result on the collision set we also use the structure of the transition probability density of $X$, and the symmetric stable “gauge” processes.

The talk is based on the joint work with R. Schilling [2].

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LARGE DEVIATIONS PROBLEM FOR RANDOM EVOLUTIONS

Volodymyr S. Koroliuk1, Igor V. Samoilenko2

Large deviation problem for a Markov random evolution process is formulated as a limit theorem for a nonlinear semigroup characterized by the exponential martingale with the exponential generator [1]

$$H^\varepsilon \phi^\varepsilon(u) := e^{-\phi^\varepsilon / \varepsilon} L^\varepsilon e^{\phi^\varepsilon / \varepsilon} \to H \varphi, \quad \phi^\varepsilon \to \varphi, \varepsilon \to 0.$$ (1)

Limit theorems for the exponential generator $H^\varepsilon, \varepsilon \to 0$ are realized for the Markov random evolution process in the schemes of asymptotically small diffusion and of the Poisson approximation.

The main method in the proof of the limit theorems is a solution of singular perturbation problem for the exponential generator on a perturbed test-functions (1).

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CONVERGENCE OF SEQUENCE OF MARKOV CHAINS TO LÉVY-TYPE PROCESS: SOLVABILITY OF THE MARTINGALE PROBLEM FOR THE LIMIT POINT AND EXPLICIT BOUNDS FOR THE CONVERGENCE RATE

T. Kosenkova

The notion of the coupling distance on the space of Lévy measures is introduced. It is proved that a martingale problem for a generator of a Lévy-type process is well-posed under the Lipschitz continuity assumption of the symbol w.r.t. the state space variable in the coupling distance. The explicit bounds for the convergence rate of a sequence of Markov chains to a Lévy-type process are given in terms of the coupling distance.

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WAVELET EXPANSIONS OF STOCHASTIC PROCESSES

Yu. V. Kozachenko

Wavelet expansions of non-random functions on $R$ were studied in different spaces. However, developed deterministic methods may not be appropriate to investigate wavelet expansions of stochastic processes. It indicates the necessity of elaborating special stochastic techniques.

We consider random processes $\xi(t)$ with $E\xi(t) = 0$ and their approximation by sums of wavelet functions, reading as follows

$$\xi_{n,k}(t) = \sum_{|k| \leq k_0} \xi_{0,k}\varphi_{0,k}(t) + \sum_{j=0}^{n-1} \sum_{|k| \leq k_j} \eta_{jk}\psi_{jk}(t),$$

where $\varphi(t)$ is an $f$-wavelet and $\psi(t)$ is an $m$-wavelet, $\varphi_{0,k}(t) = \varphi(t-k)$, $\psi_{jk}(t) = 2^{j/2}\psi(2^{j}t-k)$.

We investigate uniform convergence of $\xi_{n,k}(t)$ to $\xi(T)$ in $C(0,T)$ with probability one, in probability, in $L_p(0,T)$. We investigate the rate of convergence as well. Gaussian, $\varphi$-sub-Gaussian and random processes from an Orlicz space are considered.

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DISTRIBUTION OF THE SUPREMUM OF SOLUTION OF HOMOGENEOUS PARABOLIC EQUATION WITH INITIAL CONDITIONS FROM ORLICZ SPACE $Exp_\psi(\Omega)$

K. J. Kuchinka

We study the properties of the solution of the boundary value problem for homogeneous parabolic partial differential equation with Orlicz random initial conditions. Consider the following equation:

$$\frac{\partial}{\partial x} \left(p(x) \frac{\partial V}{\partial x}\right) - q(x)V - \rho(x) \frac{\partial V}{\partial t} = 0$$

(1)

with initial and boundary conditions

$$V(t,0) = 0, \quad V(t,\pi) = 0, \quad V(0,x) = \xi(x),$$

(2)

where $\xi(x) \in Exp_\psi(\Omega)$, such that $E\xi(x) = 0$. Let $X_k(x)$ be eigenfunctions, $\lambda_k$ be eigenvalues of the Sturm-Liouville problem.

We establish the sufficient conditions that supply the following statements:

1. The series

$$V(t,x) = \sum_{k=1}^{\infty} \xi_k e^{-\lambda_k t} X_k(x),$$

(3)

where $\xi_k = \int_0^\pi \rho(x) X_k(x) \xi(x) dx$, converges uniformly in probability for $t \in (0;T]$, $x \in [0;\pi]$. The series obtained from (3) by repeated differentiation with respect to $x$ and one differentiation with respect to $t$ converges uniformly as well.

2. The solution of the problem (1), (2) exists with probability one and can be represented in the form of the series (3).

We also obtain estimates for the distribution of the supremum of a solution of the problem (1)–(2).

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REALIZABILITY IN INFINITE AND FINITE DIMENSIONS

T. Kuna\textsuperscript{1}, M. Infusino\textsuperscript{2}, J. Lebowitz\textsuperscript{3}, E. Speer\textsuperscript{4}

In order to describe complex systems effectively, one often concentrate on a few characteristics of the system, like moments and their correlations. The large number of remaining degrees of freedom one want to suppress describing the system through effective equations just in terms of these characteristics, as for example the so-called moment closure. A related question is if an effective description of a particular type exists for a given complex system. The different characteristics are not independent of each other and not all putative choices may correspond to a complex system of a particular type. In which way one can read off the structure of the complex system from the moments? Can one construct explicitly models for the complex system? Infinite and low dimensional systems are considered.

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A LIMIT THEOREM FOR NONMARKOVIAN MULTI-CHANNEL NETWORKS IN HEAVY TRAFFIC

H. V. Livinska

We consider a multi-channel stochastic network of $[M_\rho|GI|\infty]^r$-type consisting of $r$ service nodes. An input Poisson flow of calls with the leading function $\Lambda_i(t)$ arrives at the $i$-th node of the network. Each of the $r$ nodes operates as a multi-channel stochastic system. In the $i$-th node service time is distributed with a distribution function $G_i(t)$, $i = 1, 2, ..., r$. If a call arrives in such a system then its service immediately begins. After service in the $i$-th node the call with probability $p_{ij}$ arrives to the $j$-th node of the network and with the probability $p_{rr+1} = 1 - \sum_{j=1}^{r} p_{ij}$ leaves the network, $P = ||p_{ij}||$ is a switching matrix of the network. An additional node numbered $r+1$ is interpreted as “output” from the network. At initial time the network is empty.

Let $Q_i(t), i = 1, 2, ..., r$ be a number of calls in the $i$-th node of the network at a $t$ moment of time and an $r$-dimensional process $Q(t) = (Q_1(t), ..., Q_r(t))$ will be called as a service process in the network. Our goal is to study the $r$-dimensional process $Q(t) = (Q_1(t), ..., Q_r(t))$, in conditions of heavy traffic, where $Q_i(t)$ is the number of calls in the $i$-th node at the $t$ moment of time.

It is shown that in heavy traffic the process $Q(t)$ is approximated by a Gaussian process $\xi^{(1)}(t) + \xi^{(2)}(t), t \geq 0$, where $\xi^{(i)}(t) = (\xi_{i1}(t), ..., \xi_{ir}(t)), i = 1, 2,$ two independent Gaussian processes, moreover $\xi^{(1)}(t)$ is associated with fluctuations of the input flows, and $\xi^{(2)}(t)$ – with fluctuations of service times.

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CONVERGENCE OF SEMI-MARKOV RANDOM EVOLUTIONS

I. V. Malyk

Consider semi-Markov random evolution with Markovian switching

$$\xi(t) = \xi(0) + \int_0^t \eta(ds; x(s)) + \int_0^t \gamma(ds; x(s)), t \geq 0,$$

where $x(t) \in E$ is an ergodic Markov process with generator $Q$ and stationary distribution $\pi(B), B \in \mathcal{E};$ semi-Markov process $\eta(t, x), t \geq 0, x \in E$ in $\mathcal{R}^d, d \geq 1$ generates Markov renewal process $(\eta_n, \tau_n), n \geq 0$, where $\eta_n := \eta(\tau_n, x_n), \tau_n := \int_0^t \eta(ds; x(s)),$
\[ x_n := x(\tau_n) \] with semi-Markov kernel \[ G(u, dv; x) := G(u, dv; x)F_u(t), \quad u \in \mathbb{R}_d, \, dv \in \mathbb{R}_d^d, \, t \geq 0, \, x \in E, \]

\[ G(u, dv; x) := \mathbb{P}\{\Delta \eta_{n+1} \in dv| \eta_n = u, x_n = x\}, \Delta \eta_{n+1} := \eta_{n+1} - \eta_n, \]

\[ F_u(t) := \mathbb{P}\{\theta_{n+1} \leq t| \eta_n = u\} = P(\theta_u \leq t), \, t \geq 0; \]

\[ \gamma(t; x), t \geq 0, \, x \in E \] is a diffusion process with generator \[ \Gamma(x) \varphi(u) := a(u; x)\varphi'(u) + \int_{\mathbb{R}_d^d} (\varphi(u + v) - \varphi(u) - v\varphi'(u)) \Gamma(u, dv; x). \]

For semi-Markov random evolution consider two problems: weak convergence in the average scheme and in the diffusion approximation scheme

\[ \xi^\varepsilon(t) = \xi(0) + \varepsilon^3\int_0^{t/\varepsilon^3} \eta(ds; x(\varepsilon s)) + \int_0^{t/\varepsilon^3} \gamma(ds; x(\varepsilon^2 s)) \]

\[ \xi^\varepsilon(t) = \xi(0) + \varepsilon^3\int_0^{t/\varepsilon^3} \eta(ds; x(\varepsilon s)) + \int_0^{t/\varepsilon^3} \gamma(ds; x(\varepsilon^2 s)). \]

For processes (1) and (2) we found limiting processes for \( n \to \infty \). Let \( \{\mathcal{M}_k : k \in \mathbb{N} \cup \{0\}\} \) be a Markov chain with state space \( \mathbb{N} \). Assume that \( \mathcal{M}_0 = n \) and \( \{\mathcal{M}_k\} \) is eventually nonincreasing, i.e. there exists \( a \in \mathbb{N} \) such that \( \mathbb{P}\{\mathcal{M}_{k+1} \leq \mathcal{M}_k| \mathcal{M}_k > a\} = 1 \) and \( \mathbb{P}\{\mathcal{M}_k = \mathcal{M}_{k+1}| \mathcal{M}_k > a\} < 1, \) \( k \in \mathbb{N} \cup \{0\} \). Denote by \( I_n \) the first decrement of \( \{\mathcal{M}_k\} \), namely \( I_n := n - \mathcal{M}_1 \).

We are interested in the asymptotic behaviour of the stopping time \( X_n := \inf \{k \in \mathbb{N} \cup \{0\} : \mathcal{M}_k \leq a \} \), given \( \mathcal{M}_0 = n \) as \( n \to \infty \). In this talk we provide general results concerning the weak convergence of \( X_n \) if either one of the following stationarity conditions holds true: there exists proper and non-degenerate random variable \( \xi \) such that

\[ I_n \xrightarrow{d} \xi \quad \text{as} \quad n \to \infty, \quad (1) \]

or there exists non-degenerate random variable \( \eta \) such that

\[ \frac{I_n}{n} \xrightarrow{d} 1 - \eta \quad \text{as} \quad n \to \infty. \quad (2) \]

Our approach is based on the observation that in both cases (1) and (2) the sequence \( \{X_n\} \) can be approximated by a suitable renewal counting process and the error of such an approximation is estimated in terms of an appropriate probability distance.

This method has been used in [1] to derive a weak convergence result for the number of collisions in beta-coalescents and in [2] to prove a central limit theorem for the number of zero increments in the random walk with a barrier.

\[ \frac{X_n}{n} \xrightarrow{d} \xi \quad \text{as} \quad n \to \infty, \quad (1) \]

or there exists non-degenerate random variable \( \eta \) such that

\[ \frac{X_n}{n} \xrightarrow{d} 1 - \eta \quad \text{as} \quad n \to \infty. \quad (2) \]

\[ \frac{X_n}{n} \xrightarrow{d} \frac{1}{\eta} \quad \text{as} \quad n \to \infty. \quad (2) \]

Our approach is based on the observation that in both cases (1) and (2) the sequence \( \{X_n\} \) can be approximated by a suitable renewal counting process and the error of such an approximation is estimated in terms of an appropriate probability distance.

This method has been used in [1] to derive a weak convergence result for the number of collisions in beta-coalescents and in [2] to prove a central limit theorem for the number of zero increments in the random walk with a barrier.

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\[ \frac{X_n}{n} \xrightarrow{d} \frac{1}{\eta} \quad \text{as} \quad n \to \infty. \quad (2) \]
In order to obtain these results the methods of random processes from the space $F_u(\Omega)$ have been used (see [1]). Space $F_u(\Omega)$ consists of such random variables $\xi$ that $\sup_{u \geq 1} \frac{|\mathbb{E}[\xi^u]|^{1/u}}{\psi(u)} < \infty$, where $\psi(u) > 0$, $u \geq 1$ is the monotonically increasing and continuous function for which $\psi(u) \to \infty$ as $u \to \infty$.

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ANALOGUE OF THE BERRY-ESSEEN THEOREM FOR FUNCTIONALS OF WEAKLY ERGODIC MARKOV PROCESS

G. M. Molyboga

Bounds for the rate of convergence in the central limit theorem for functionals of weakly ergodic Markov processes are obtained. The method of proof generalizes the one proposed in [1, 2], aimed to prove diffusion approximation type theorems for systems with weakly ergodic Markov perturbations.

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THE INTEGRAL EQUATION FOR THE FOURIER-STIELTJES TRANSFORM OF THE SEMI-MARKOV RANDOM WALK WITH POSITIVE TENDENCY AND NEGATIVE JUMPS

T. H. Nasirova$^1$, S. Tunc Yavuz$^2$, U. C. Idrisova$^3$

Let the sequence of i.i.d. random variables $\{\xi_k, \zeta_k\}_{k=1}^{\infty}$ be defined on the probability space $(\Omega, F, P (\cdot))$. Random variables $\xi_k$ and $\zeta_k$ are mutually independent. We construct the process

$$X(t) = z + t - \sum_{i=1}^{k} \zeta_i, \quad \text{if} \quad \sum_{i=0}^{k} \xi_i \leq t < \sum_{i=0}^{k+1} \xi_i, \quad k = 0, 1, 2, \ldots,$$

using random variables $\xi_k$ and $\zeta_k$. This process is called the semi-Markov random walk with positive tendency and negative jumps. Let

$$R(t, x | z) = P \{X(t) < x | X(0) = z\}, \quad x \in R,$$

$$\tilde{R}(\theta, x | z) = \int_{t=0}^{\infty} e^{-\theta t} R(t, x | z), \quad \theta > 0,$$

$$\tilde{R}(\theta, \gamma | z) = \int_{x=-\infty}^{\infty} e^{\gamma x} d_x \tilde{R}(\theta, x | z), \quad \gamma \in R.$$

Theorem 1. If $\{\xi_k, \zeta_k\}_{k=1}^{\infty}$ is the sequence of i.i.d. positive random variables, then

$$\tilde{R}(\theta, \gamma | z) = \int_{x=-\infty}^{\infty} e^{\gamma x} d_x \tilde{R}(\theta, x | z), \quad \gamma \in R,$$

satisfies the following integral equation

$$\tilde{R}(\theta, x | z) = \varepsilon (x - z) \int_{t=0}^{x-z} e^{-\theta t} P \{\xi_1 > t\} dt -$$

$$- \int_{y=-\infty}^{z} \tilde{R}(\theta, x | y) \int_{t=0}^{\infty} e^{-\theta t} d_y P \{\xi_1 < z + t - y\} dP \{\xi_1 < t\} -$$

$$- \int_{y=z}^{\infty} \tilde{R}(\theta, x | y) \int_{t=0}^{\infty} e^{-\theta t} d_y P \{\xi_1 < z + t - y\} dP \{\xi_1 < t\}.$$

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RANDOM FLIGHTS

Enzo Orsingher

In this talk we present different types of random flights and examine their dynamics, probability laws and governing equations. We first consider the telegraph process (a random flight on the line), discuss its distribution, the connections with the telegraph equations, the first-passage time and the limiting case. We consider also the asymmetric telegraph process and its reduction to the symmetric one by means of relativistic transformations.

Planar motions with a finite number of directions (in particular, four orthogonal directions) and an infinite number of directions, chosen at Poisson times with uniform law, are examined and several explicit distributions derived. A particular attention is devoted to the second model, where conditional and unconditional distributions are presented and the related equation of damped planar vibrations probabilistically derived.

Random flights in $\mathbb{R}^d$ are subsequently considered and characteristic functions of the position of moving particles performing the random flights, obtained. The cases $d=2$, $d=4$, are investigated in detail.

We present random flights in $\mathbb{R}^d$ with Dirichlet joint distribution for displacements, hyperspherical uniform law for the orientation of steps and with a fractional Poisson number of changes of direction.

Two types of fractional extensions of the above material are presented. The first one is obtained by considering Dzherbashyan-Caputo types of time-fractional derivatives. The second fractional extension is obtained by considering fractionalisation of Klein-Gordon equations and by applying the McBride approach to fractional powers of Bessel-type operators.

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ACCURACY OF SIMULATION OF THE GENERALIZED WIENER PROCESS

A. A. Pashko

The paper is a continuation of [1]. This work investigates the estimation of accuracy of simulation of generalized Wiener process.

Let $(T, \Sigma, \mu)$ be some measurable space. Generalized Wiener process $W_\alpha(t)$ with parameter $\alpha \in (0, 2]$ can be represented as

$$W_\alpha(t) = \frac{A}{\sqrt{\pi}} \left( \int_0^\infty \cos(\lambda t) - \frac{1}{\lambda^{\alpha+1}} d\xi(\lambda) - \int_0^\infty \frac{\sin(\lambda t)}{\lambda^{\alpha+1}} d\eta(\lambda) \right),$$

where $\xi(\lambda), \eta(\lambda)$ are independent real standard Wiener processes, $E\xi(\lambda) = E\eta(\lambda) = 0$, and $E(d\xi(\lambda))^2 = E(d\eta(\lambda))^2 = d\lambda$;

$$A^2 = \left( \frac{2}{\pi} \int_0^\infty \frac{1 - \cos(\lambda t)}{\lambda^{\alpha+1}} d\lambda \right)^{-1} = \left( -\frac{2}{\pi} \Gamma(-\alpha) \cos \left( \frac{\alpha \pi}{2} \right) \right)^{-1}.$$

Let $D_M : 0 = \lambda_0 \leq \lambda_1 \leq \ldots \leq \lambda_M = \Lambda$ be a partition of the set $[0, \Lambda]$. The process simulation is

$$S_M(t, \Lambda) = \frac{A}{\sqrt{\pi}} \left( \sum_{i=1}^M \frac{\cos(\lambda_i t)}{\lambda_i^{\alpha+1}} X_i + \sum_{i=1}^M \frac{\sin(\lambda_i t)}{\lambda_i^{\alpha+1}} Y_i \right),$$

where $\{X_i, Y_i\}$ are independent Gaussian random variables, $EX_i = EY_i = 0$ and $EX_i^2 = EY_i^2 = \lambda_i - \lambda_{i-1}$.

**Theorem 1.** Model $S_M(t, \Lambda)$ approximates process $W_\alpha(t)$ with accuracy $\delta > 0$ and reliability $\varepsilon, 0 < \varepsilon < 1$ in $L_2(T)$ if inequalities hold $\delta > B(M)$ and

$$\exp \left\{ \frac{1}{2} \int_T E(W_\alpha(t) - S_M(t, \Lambda))^2 dt \right\} \leq 1 - \varepsilon,$$

where $B(M) = \left( \int_T E(W_\alpha(t) - S_M(t, \Lambda))^2 dt \right)^{\frac{1}{2}}$.

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ON THE STRONG EXISTENCE AND UNIQUENESS TO AN SDE THAT DESCRIBES COUNTABLE INTERACTING PARTICLE SYSTEM

Andrey Pilipenko, Maksym Tantsiura

A stochastic differential equation describing the motion of a countable system of interacting particles is considered:

$$\left\{ \begin{array}{l}
    dX_k(t) = a(X_k(t), \mu_t) dt + b(X_k(t), \mu_t) dw_k(t), \quad k \in \mathbb{Z}, t \in [0, T], \\
    \mu_t = \sum_{k \in \mathbb{Z}} \delta X_k(t), \\
    X_k(0) = u_k, \quad k \in \mathbb{Z}, 
\end{array} \right.$$
where \( a \) is a bounded measurable function, \( b \) is a bounded Lipschitz function. A theorem on existence and uniqueness of the strong solution is proved if \( a \) and \( b \) satisfy finite interaction radius condition.

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GROWTH OF ANALYTIC IN THE CIRCLE CHARACTERISTIC FUNCTIONS OF PROBABILITY LAWS

M. I. Platsydem\(^1\), O. Kinash\(^2\)

Let \( F \) be a probability law (or a distribution function). The function \( \varphi(z) = \int_\infty e^{izx} dF(x) \), defined for real \( z \) is a characteristic function of this law.

It is known that \( \varphi \) is analytic in \( \mathbb{D}_R = \{ z : |z| < R \} , 0 < R \leq +\infty \), characteristic function if and only if for every \( r \in (0, R) \)

\[
W_F(x) := 1 - F(x) + F(-x) = O \left( e^{-rx} \right) , x \to +\infty .
\]

Let \( L \) be a class of continuous increasing functions \( \alpha \) such that \( \alpha(x) \equiv \alpha(x_0) > 0 \), as \( x \leq x_0 \) and \( \alpha(x) \) increases to \( +\infty \) as \( x \leq x_0 \to +\infty \). We say that \( \alpha \in L_{\text{si}} \) if \( \alpha \in L \) and \( \alpha(x) = (1 + o(1)) \alpha(x) \) as \( x \to +\infty \) for any \( c \in (0, +\infty) \), that is \( \alpha \) is slowly increasing function.

**Theorem 1.** Let function \( \alpha \in L_{\text{si}} \) such that \( \alpha(\ln x) \to 0 \) and \( \alpha \left( \frac{x}{\alpha(\alpha(1))} \right) = (1 + o(1)) \alpha(x) \) as \( x \to +\infty \) for any \( c \in (0,1) \), and \( \varphi \) be an analytic in \( \mathbb{D}_R \), \( 0 < R < \infty \), characteristic function of the probability law \( F \), which satisfies the condition \( \lim_{r \uparrow R} W_F(x)^eRx = +\infty \). Let \( \rho^\alpha_\varphi \left[ \varphi \right] = \lim_{r \uparrow R} \alpha(\ln M(\varphi |x|)) \).

1. If \( \rho^\alpha_\varphi \left[ \varphi \right] \geq 1 \), then \( \rho^\alpha_\varphi \left[ \varphi \right] = \lim_{x \to +\infty} \alpha \left( \frac{\ln(\varphi(x))}{\alpha(x)} \right) \).
2. If \( \rho^\alpha_\varphi \left[ \varphi \right] < 1 \), then \( \rho^\alpha_\varphi \left[ \varphi \right] = \lim_{x \to +\infty} \alpha \left( \frac{\ln(W_F(x)^eRx)}{\alpha(x)} \right) \).

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**COMPARISON OF DIFFERENT BASES FOR WAVELET EXPANSIONS OF GAUSSIAN RANDOM PROCESSES**

O. V. Polosmak

The most actual issues of application of wavelet analysis related with signal processing and simulation, audio and image compression, noise removal, the identification of short-term and global patterns, spectral analysis of the signal. Wavelet representations could be used to convert the problem of analyzing a continuous-time random process to that of analyzing a random sequence, which is much simpler. So wavelets expansions of random processes are studied. We investigate uniform convergence of wavelet expansions of Gaussian random processes. The convergence is obtained under simple general conditions on processes and wavelets which can be easily verified. Applications of the developed technique are shown for several wavelet bases. So, conditions of uniform convergence for Battle-Lemarie wavelets and Meyer wavelets expansions of Gaussian random processes are presented. Another useful in various computational applications thing is the rate of convergence, especially if we are interested in the optimality of the stochastic approximation or the simulations. An explicit estimate of the rate of uniform convergence for Battle-Lemarie wavelets and Meyer wavelets expansions of Gaussian random processes is obtained and compared.

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**ON THE CROSS-CORRELOGRAM ESTIMATOR OF THE RESPONSE FUNCTION**

I. V. Rozora

A time-invariant casual continuous linear Volterra system with response function \( H(\tau) , \tau \in \mathbb{R} \), is considered. It means that the real-valued function \( H(\tau) = 0 \) as \( \tau < 0 \), and the response of the system to an input process \( X(t) , t \in \mathbb{R} \) has such form

\[
Y(t) = \int_0^\infty H(\tau)X(t-\tau)d\tau.
\]

(1)
One of the problems arising in the theory of such systems is to estimate or identify the function $H$ by observations of the responses of the system. Here we use a method of correlograms for the estimation of the response function $H$.

Assume that $X = (X(t), t \in \mathbb{R})$ is a measurable real-valued stationary zero-mean Gaussian process with known spectral density. The reaction of the system on an input signal $X$ is represented by (1).

The so-called cross-correlogram (or sample cross-correlogram function)

$$
\hat{H}_T(\tau) = \frac{1}{T} \int_0^T Y(t + \tau)X(t)dt, \tau > 0
$$

will be used as an estimator for $H$. Here $T$ is the length of the averaging interval.

The inequality of large deviation probability for $\hat{H}_T(\tau) - H(\tau)$ in uniform norm is founded. The theory of square-Gaussian processes is used.

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ON THE KOLMOGOROV-MARCINKIEWICZ-ZYGMUND TYPE SLLN FOR ELEMENTS OF REGRESSION SEQUENCES

M. K. Runovska

This talk arose due to the author’s inspiration for recent works by F. Hechnier, B. Heinzel [1], and Deli Li, Yongcheng Qi, A. Rosalsky [2] on the Kolmogorov-Marcinkiewicz-Zygmund type SLLN for i.i.d. random variables and i.i.d. random Banach-valued elements. In this work related results for elements of regression sequences of random variables are obtained. Thus, consider a zero-mean linear regression sequence of random variables $(\xi_k) = (\xi_k, k \geq 1)$:

$$
\xi_1 = \eta_1, \xi_k = \alpha_k \xi_{k-1} + \eta_k, k \geq 2,
$$

(1)

where $(\alpha_k)$ is a nonrandom real sequence, and $(\eta_k)$ is a sequence of independent symmetric random variables. Set $S_n = \sum_{k=1}^n \xi_k, n \geq 1$, and study necessary and sufficient conditions providing almost sure convergence of the series

$$
\sum_{n=1}^\infty \frac{S_n}{n^{1+p}},
$$

(2)

where $p > 0$. Let us formulate one of obtained results.

Theorem 1. Let in (1) $\alpha_k = \alpha = \text{const}$, for any $k \geq 2$, and $(\eta_k)$ be a sequence of independent copies of a symmetric random variable $\eta$. The series (2) converges a.s. iff the following two conditions hold:

- $-1 < \alpha < 1$ and $0 < p < 2$, or $\alpha = 1$ and $0 < p < 1$,
- $E|\eta|^p < \infty$.

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RATE FUNCTIONAL IN A LARGE DEVIATIONS PROBLEM FOR MARKOV PROCESSES

Igor V. Samoilenko

The problem of large deviations is solved in four stages ([1], Ch.2):

1) Verification of convergence of the exponential (nonlinear) generator $H^\epsilon$, to the limit exponential (nonlinear) generator $H$;

2) Verification of exponential tightness of the pre-limit Markov processes. Convergence of the semigroups, corresponding to $H^\epsilon$, and exponential tightness of the pre-limit Markov processes give the large deviations principle in $D_{\mathbb{E}}[0, \infty]$;

3) Verification of the comparison principle for the limit exponential generator, showing that the semigroups, corresponding to $H^\epsilon$ really converge to the unique semigroup, corresponding to $H$;

4) Construction of a variational representation for the limit exponential generator that gives the rate functional.

The limit theorems for the Markov processes (MP) are based on a martingale characterization of MP $\eta(t), t \geq 0$:

$$
\mu(t) := \varphi(\eta(t)) - \varphi(\eta(0)) - \int_0^t L\varphi(\eta(s))ds
$$

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with the generator
\[ L\varphi(u) := \lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(u + \Delta \eta(t)) - \varphi(u)|\eta(t) = u], \quad \Delta \eta(t) := \eta(t + \Delta) - \eta(t). \]

By analogy, the LDP for the Markov processes (MP) is based on the exponential martingale characterization of MP \( \eta(t), t \geq 0 \):
\[ \mu_c(t) := exp\{\varphi(\eta(t)) - \varphi(\eta(0)) - \int_0^t H\varphi(\eta(s))ds\} \]

with the exponential generator (EG)
\[ H\varphi(u) := e^{-\varphi(u)} L e^{\varphi(u)}. \]

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M. N. Savchuk

FUNCTIONAL LIMIT THEOREM FOR A NON-EQUIPROBABLE ALLOCATION OF PARTICLES BY SERIES

The arrangement of particles by series on an orderly and numbered by natural numbers cells is considered. Allocation of particles in different series is independent. In each series particles are placed in a cell with no more than one in each and in such a way that the probability of placing the particle in a specific cell is a function of cell number and the total number of particles caught in the previous cell. Vector random process is constructed that described one in each and in a such way that the probability of placing the particle in a specific cell is a function of cell number.

REFERENCES


V. I. Senin

ON THE APPROXIMATE HÖLDER INDEX FOR TRAJECTORIES OF STABLE PROCESSES

For almost all trajectories of the symmetric \( \alpha \)-stable process (\( \alpha < 2 \)) the following property is proved: for any \( \gamma \) with \( \alpha \gamma < 1 \) and any \( \epsilon > 0 \), there exists a Hölder continuous function with exponent \( \gamma \) which coincides with the trajectory up to a set of Lebesgue measure \( \leq \epsilon \).

V. I. Senin

ASSESSMENT AND OPTIMAL POLICIES OF SEMI-CONTINUOUS KILLED MARKOV DECISION PROCESSES

P. R. Shpak\(^1\), Ya. I. Yeleyko\(^2\)

We consider semi-continuous killed Markov decision processes [1] which satisfies following conditions:

1. Set of states \( X \) and set of actions \( A \) are separable metric spaces.
2. If \( x_k \to x \in X \) and \( a_k \in A(x_k) \) then there exists a limit point of sequence \( \{a_k\} \), which belongs to set \( A(x) \).
3. If \( f \in L(X_i) \) and \( g(a) = \int \mu(dx) l f(x) (a \in A_t) \) then \( g \in L(A_t) \) (\( t = m + 1,...,n \)) (where \( L(E) \) is the set of all semi-continuous functions on \( E \) [2]).
4. Reward function \( q \) belongs to \( \mathcal{L}(A_t) \), crash function \( c \) belongs to \( \mathcal{L}(X_i \cap X^*) \) and terminal reward \( r \) belongs to \( \mathcal{L}(X_i) \).

We show that following theorem takes place for semi-continuous killed Markov decision process.

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Theorem 1. For semi-continuous killed Markov decision process following conditions are satisfied:
1. Assessment of process \( \nu \) belongs to \( L(X_m) \).
2. \( \nu(\mu) = \mu \nu \) for arbitrary initial distribution \( \mu \).
3. Optimal policy exists.

References

Gaussian Fredholm Processes

Tommi Sottinen\(^1\), Lauri Viitasaari\(^2\)

We show that every separable Gaussian process with integrable variance function admits a Fredholm representation with respect to a Brownian motion. We extend the Fredholm representation to a transfer principle and develop stochastic analysis by using it. In particular, we prove an Itô formula that is, as far as we know, the most general Malliavin-type Itô formula for Gaussian processes so far. Finally, we give applications to equivalence in law and series expansions of Gaussian processes.

Our key theorem is

**Theorem 1.** Let \( X = (X_t)_{t \in [0,T]} \) be a separable centered Gaussian process. Then there exists a kernel \( K_T \in L^2([0,T]^2) \) and a Brownian motion \( W = (W_t)_{t \geq 0} \), independent of \( T \), such that
\[
X_t = \int_0^T K_T(t,s) \, dW_s
\]
if and only if the covariance \( R \) of \( X \) satisfies the trace condition
\[
\int_0^T R(t,t) \, dt < \infty.
\]

The talk is based on the preprint [1].

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Existence and Uniqueness of Invariant Measures for Stochastic Reaction-Diffusion Equations in Unbounded Domains

O. M. Stanzhitskyi

We study the long-time behavior of stochastic reaction-diffusion equations of the type
\[
du = (Au + f(x, u)) \, dt + \sigma(x, u) \, dW(t, x),
\]
where \( A \) is an elliptic operator, \( f \) and \( \sigma \) are nonlinear maps and \( W \) is an infinite dimensional nuclear Wiener process. The emphasis is on unbounded domains. Under the assumption that the nonlinear function \( f \) possesses certain dissipative properties, this equation is known to have a solution with an expectation value which is uniformly bounded in time. Together which some compactness property, the existence of such a solution implies the existence of invariant measure which is important step in establishing the ergodic behavior of underlying physical systems. We expand the existing classes of nonlinear functions \( f \) and \( \sigma \) and elliptic operator \( A \) for which the invariant measure exists, in particular, in unbounded domains.

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CONSTRUCTION MODEL OF GAUSSIAN HOMOGENEOUS AND ISOTROPIC STOCHASTIC FIELD

N. V. Troshki

The main objective of the work is to construct the model of Gaussian homogeneous and isotropic stochastic field which approximates it in \( L_p(T), p \geq 1 \) with a given accuracy and reliability. Let \( X = \{X(t, x), t \in \mathbb{R}, x \in [0, 2\pi]\} \) be a real-valued Gaussian homogeneous and isotropic stochastic field, which has the following representation [1]

\[
X(t, x) = \sum_{k=1}^{\infty} \cos(kx) \int_{0}^{\infty} J_k(t\lambda)d\eta_{1,k}(\lambda) + \sum_{k=1}^{\infty} \sin(kx) \int_{0}^{\infty} J_k(t\lambda)d\eta_{2,k}(\lambda),
\]

where \( \eta_{i,k}(\lambda), i = 1,2, k = 1,\infty \) are independent Gaussian processes with independent increments, \( E\eta_{i,k}(\lambda) = 0, E(\eta_{i,k}(b) - \eta_{i,k}(c))^2 = F(b) - F(c), b > c, F(\lambda) \) is a spectral function of the field and \( J_k(u) \) are Bessel’s integrals. Let \( \Lambda > 0 \) be a number. We consider such partition \( L = \{\lambda_0, ..., \lambda_N\} \) of the set \([0, \infty]\) that \( \lambda_0 = 0, \lambda_k < \lambda_{k+1}, \lambda_{N-1} = \Lambda, \lambda_N = +\infty \). As a model for the \( X(t, x) \) we consider

\[
\hat{X}(t, x) = \sum_{k=1}^{M} \cos(kx) \sum_{l=0}^{N-1} \eta_{1,k, l}J_k(t\xi_l) + \sum_{k=1}^{M} \sin(kx) \sum_{l=0}^{N-1} \eta_{2,k, l}J_k(t\xi_l),
\]

where \( \eta_{i,k, l} = \int d\eta_{i,k}(\lambda), i = 1,2 \) are such independent Gaussian random variables that \( E\eta_{i,k, l} = 0, E\eta_{i,k, l}^2 = F(\lambda_{i+1}) - F(\lambda_i) = b_i^2 \), \( \xi_l \) are independent random variables taking values on the segments \([\lambda_l, \lambda_{l+1}]\) and \( F(\lambda) = P\{\xi < \lambda\} = \frac{\int_{\lambda}^{\lambda_{i+1}} F(t)dt}{\int_{\lambda}^{\lambda_{i+1}} F(t)dt} \).

We also found conditions under which this model approximates the field \( X(t, x) \) with a given accuracy and reliability in the space \( L_p(T), p \geq 1 \).

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RESTRICTED ISOMETRY PROPERTY FOR SOME RANDOM MATRIX

V. B. Troshki

Nowadays a new approach to the signal processing that called compressive sensing is actively developing. Modern development of this theory started with the works of Candes and Tao [2] and Donoho [3]. But the problem of constructing the universal matrix of measurements is not solved yet. Let \( \Sigma_K \) be a set of all vectors from \( \mathbb{R}^N \) which contain not more as \( K \) non-zero elements. We say that a matrix \( A \) satisfies the Restricted Isometry Property (RIP) of order \( K \) if there exists a \( \delta \in (0; 1) \) such that

\[
(1 - \delta) \|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta) \|x\|_2^2
\]

holds for all \( x \in \Sigma_K \).

Let \( U_{\alpha \xi} = \exp\{i x |\xi|\} \), \( 0 < \alpha \leq 1 \) and \( S_{U_{\alpha \xi}}(\Omega) \) is a family of random variables, where for each \( \xi \) there exists a constant \( r \) such that \( EU_{\alpha \xi}(\xi) < \infty \). We propose to use a matrix the elements of which are random variables from \( S_{U_{\alpha \xi}}(\Omega) \). Note that \( S_{U_{\alpha \xi}}(\Omega) \) and Orlicz space \( L_{u}(\Omega) \) are equivalent. Some information about the Orlicz space you can find in book [1]. Also we established, that the matrix with such elements satisfies the restricted isometry properties, which is one of the main concepts in the theory of compressive sensing.

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A MULTIPLICATIVE WAVELET-BASED REPRESENTATION FOR A RANDOM PROCESS

I. Turchyn

Let \( \phi \) be a father wavelet, \( \psi \) – the corresponding mother wavelet. We consider random processes \( Y(t) \), where

\[
Y(t) = \exp\{X(t)\},
\]

\( X = \{X(t), \ t \in \mathbb{R}\} \) is a centered second-order random process which correlation function can be represented as

\[
R(t,s) = \int_{\mathbb{R}} u(t,y) \overline{u(s,y)} \, dy,
\]

\( u(t,\cdot) \in L_2(\mathbb{R}) \).

We study the representation

\[
Y(t) = \prod_{k \in \mathbb{Z}} \exp\{\xi_{0k}a_{0k}(t)\} \prod_{j=0}^{\infty} \prod_{l \in \mathbb{Z}} \exp\{\eta_{jl}b_{jl}(t)\},
\]

(1)

where

\[
a_{0k}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \phi_{0k}(y) \, dy,
\]

\[
b_{jk}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \psi_{jk}(y) \, dy,
\]

the random variables \( \xi_{0k}, \eta_{jk} \) are centered and uncorrelated. The rate of convergence of product (1) is found for the case when \( X(t) \) is a strictly sub-Gaussian process.

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SIMULATION OF \( \text{Sub}_\varphi(\Omega) \)-PROCESSES

O. I. Vasylyk

An algorithm for simulation of \( \text{Sub}_\varphi(\Omega) \)-processes that are weakly self-similar with stationary increments in the sense that they have the covariance function \( R(t,s) = \frac{1}{2} \left( t^{2H} + s^{2H} - |t-s|^{2H} \right) \), where \( H \in (0,1) \), was presented in the paper [1]. In order to construct a model of such a process we used a series expansion approach.

Let us recall that a zero mean random variable \( \xi \) belongs to the space \( \text{Sub}_\varphi(\Omega) \), the space of \( \varphi \)-sub-Gaussian random variables, if there exists a positive and finite constant \( a \) such that the inequality \( \mathbb{E}\exp\{\lambda \xi\} \leq \exp\{\varphi(a\lambda)\} \) holds for all \( \lambda \in \mathbb{R} \). Here \( \varphi \) is an Orlicz N-function and is quadratic around the origin, i.e. there exist such constants \( x_0 > 0 \) and \( C > 0 \) that \( \varphi(x) = Cx^2 \) for \( |x| \leq x_0 \).

A stochastic process \( X = \{X_t\}_{t \in [0,1]} \) is a \( \text{Sub}_\varphi(\Omega) \)-process if it is a bounded family of \( \text{Sub}_\varphi(\Omega) \) random variables: \( X_t \in \text{Sub}_\varphi(\Omega) \) for all \( t \in [0,1] \) and \( \sup_{t \in [0,1]} \varphi(x_t) < \infty \).

Processes of fractional Brownian motion belong to the space \( \text{Sub}_\varphi(\Omega) \) with function \( \varphi(x) = x^2/2 \). Some examples of simulation of fractional Brownian motion for different values of parameter \( H \) with given reliability and accuracy in space \( C([0,1]) \) were considered in the paper [2].

Optimization of some model parameters and illustration of performance of the simulation algorithm in the case of fractional Brownian motion will be presented at the conference PRESTO-2015.

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A GENERAL APPROACH TO SMALL DEVIATION VIA CONCENTRATION OF MEASURES

L. Viitasaari¹, E. Azmoodeh²

Let \( y = (y_t)_{t \in T} \) be a stochastic processes with some index set \( T \), and let \( \| \cdot \| \) be a norm. The small deviation problem refers to analysing the probability \( \mathbb{P}(\|y\| < \epsilon) \) for some given small number \( \epsilon > 0 \). In particular, in many cases one is interested in finding a good upper bound for the small deviation probability, and one is interested in the supremum norm \( \| \cdot \|_\infty \).

General small deviation problems have received a lot of attention recently due to their connections to various mathematical topics as well as importance for various applications. Similarly, large deviation theory and concentration of measure phenomena play important role in various topics in mathematics as well as in applications. In general the theory of large deviation and it link to the concentration of measure is better understood than the theory of small deviations. Indeed, the small deviation problems are usually studied only in some particular cases. For example, Gaussian processes with stationary increments and related processes have received a lot of attention. However, while the problem is well-studied in some special cases, it seems there does not exist a unified approach to attack the problem in full generality covering all kind of processes.

In this talk we introduce a general approach to find upper bounds for small deviation probabilities which reveals the connection of small deviation theory to the concentration of measure phenomena; an extensively studied and important topic which is also closely related to large deviation theory. The advantages of the presented general approach is that it does not rely on any assumptions of the underlying process \( y \) a priori, and it can be used to study different norms. The power of the approach is illustrated by showing how it can be used to recover some optimal results in some well-studied particular examples.

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SOME PROPERTIES OF A QUEUE WITH \( \varphi \)-SUB-GAUSSIAN INPUT

Rostyslav Yamnenko

We study a workload process of a general single server queue

\[ A(t) = mt + \sigma X(t), \quad t \geq 0, \]

where \( m > 0 \) and \( \sigma > 0 \) are some constants, \( X \) is an input \( \varphi \)-sub-Gaussian random process, e.g. a generalized \( \varphi \)-sub-Gaussian fractional Brownian motion. Recall that a centered random variable \( \xi \) belongs to the space of \( \varphi \)-sub-Gaussian random variables, if \( \mathbb{E}\exp(\lambda \xi) \) exists for all \( \lambda \in \mathbb{R} \) and there exists a positive constant \( a \) such that the following inequality

\[ \mathbb{E}\exp(\lambda \xi) \leq \exp(\varphi(a\lambda)) \]

holds for all \( \lambda \in \mathbb{R} \).

The class of \( \varphi \)-sub-Gaussian stochastic processes is more general than the Gaussian one [1]; therefore, all results obtained in general case are valid for Gaussian processes for certain Orlicz \( \varphi \)-function \( \varphi \).

We present the upper bound of the buffer overflow probability

\[ \mathbb{P}\left\{ \sup_{a \leq t \leq b} (A(t) - ct) > x \right\} \]

for several queuing models defined on any finite time interval \([a, b]\), where \( c > m \) is a constant output rate.

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THE BEHAVIOUR OF GENERATOR NORMALIZATION FACTOR IN APPROXIMATION OF RANDOM PROCESSES

O. A. Yarova¹, Ya. I. Yeleyko²

Research of different types of approximations and Markov random evolutions was founded by Koroliuk [1], Feng [1] and others. In particular, Feng studied the problem of large deviations for processes with independent increments in Poisson approximation schemes and Lévy approximation. In these approximations generators are normalized by linear factor. However, in some cases, this normalization is not appropriate. Therefore we need to consider normalizing factor as a nonlinear function. The purpose of this work is to find parameters that normalize the generator of random process with independent increments.

Consider the generator of process with independent increments in Poisson approximation scheme

\[ \Gamma^\varepsilon \varphi(u) = (g_1(\varepsilon))^{-1} \int_R (\varphi(u + v) - \varphi(u)) \Gamma^\varepsilon(dv), \]

where \( g_1(\varepsilon) \rightarrow 0 \) when \( \varepsilon \rightarrow 0 \), initial conditions are \( b_\varepsilon = g_1(\varepsilon)(b + \theta_\varepsilon^c) \), \( c_\varepsilon = g_1(\varepsilon)(c + \theta_\varepsilon^c) \), \( \Gamma^\varepsilon_g = g_1(\varepsilon)(\Gamma_g + \theta_\varepsilon^\varepsilon^c) \).

Then this generator has the following asymptotic representation

\[ \Gamma^\varepsilon \varphi(u) = b\varphi(u) + \int_R (\varphi(u + v) - \varphi(u) - v\varphi'(u)) \Gamma^0(dv) + \theta^\varepsilon \varphi, \]

where \( |\theta^\varepsilon \varphi| \rightarrow 0 \).

Now consider Levy approximation. The generator of this approximation is normalized by parameter \( g_2(\varepsilon) \), where \( g_2(\varepsilon) = o(g_1(\varepsilon)) \), and has the form

\[ \Gamma^\varepsilon \varphi(u) = (g_2(\varepsilon))^{-2} \int_R (\varphi(u + v) - \varphi(u)) \Gamma^\varepsilon(dv), \]

where \( g_2(\varepsilon) \rightarrow 0 \) when \( g_1(\varepsilon) \rightarrow 0 \).

The generator of Levy approximation has the next asymptotic representation

\[ \Gamma^\varepsilon \varphi(u) = g_1(\varepsilon)^{-1} b \varphi(u) + (b - b_\varepsilon) \varphi'(u) + \frac{c - c_\varepsilon}{\varepsilon} \varphi^n(u) + \int_R (\varphi(u + v) - \varphi(u) - v\varphi'(u)) \Gamma^0(dv) + \theta^\varepsilon \varphi. \]

This normalization allows to evaluate the jumps in approximation processes with independent increments and to find their asymptotic representation.

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LIPSCHITZ CONDITIONS FOR RANDOM PROCESSES FROM \( L_p(\Omega) \) SPACES OF RANDOM VARIABLES

D. V. Zatula

We study Lipschitz continuity of random processes \( X = (X(t), t \in T) \) from \( L_p(\Omega) \) spaces and provide estimates for the distribution of sample paths of such processes.

Let \((T, \rho)\) be some metric space. We study conditions under which sample paths of random processes \( X = (X(t), t \in T) \) satisfy a Lipschitz condition. In particular, we consider a function \( f \) such that the following inequality holds:

\[ \limsup_{\varepsilon \downarrow 0} \frac{\sup_{0 < \rho(t, s) \leq \varepsilon} |X(t) - X(s)|}{f(\varepsilon)} \leq 1. \]

This function is a modulus of continuity for the random process \( X \) from \( L_p(\Omega) \) space of random variables. The main interest for us is to estimate probabilities

\[ P \left\{ \sup_{0 < \rho(t, s) \leq \varepsilon} \frac{|X(t) - X(s)|}{f(\rho(t, s))} > x \right\}. \]

For Gaussian processes similar results were obtained by Dudley [1]. These results were generalized for some classes of processes from Orlitz spaces by Buldygin and Kozachenko [2].

There are applications of Lipschitz continuity of random processes to the study of the rate of approximation of functions by trigonometric polynomials.
ON THE ASYMPTOTICS OF RANDOM SUMS

N. M. Zinchenko

Let \( \{X_i, i \geq 1\} \) be i.i.d., \( E[X_1] = m \), \( S(t) = \sum_{i=1}^{[t]} X_i \), \( t > 0 \), \( S(0) = 0 \). Also suppose that \( \{Z_i, i \geq 1\} \) is another sequence of non-random i.i.d.r.v. independent of \( \{X_i, i \geq 1\} \) with \( E[Z_1] = 1/\lambda > 0 \), \( Z(x) = \sum_{i=1}^{[x]} Z_i \), \( x > 0 \), \( Z(0) = 0 \), and define the renewal (counting) process \( N(t) = \inf\{x \geq 0 : Z(x) > t\} \). Random sums (randomly stopped sums) are defined as \( D(t) = S(N(t)) = \sum_{i=1}^{N(t)} X_i \), where i.i.d.r.v. \( \{X_i, i \geq 1\} \) and renewal process \( N(t) \) are given above.

We proposed a number of integral tests for investigation the rate of growth of the random sums \( D(t) \) as \( t \to \infty \) and the asymptotic behavior of increments \( D(t + a_t) - D(t) \) on intervals, whose length \( a_t \) grows, but not faster than \( t \). For instance, when both \( \{X_i\} \) and \( \{Z_i\} \) have moments of order \( p \geq 2 \), then non-decreasing function \( f(t) = ct^{1/2} h(t) \), \( h(t) \uparrow \infty \), \( c > 0 \), will be an upper (lower) function for centered process \( (D(t) - mt) \) according to convergence (divergence) of the integral \( \int_t^{\infty} t^{-1} h(t) \exp(-h^2(t)/2) dt \). Corresponding proofs are based on the rather general theorems about the strong approximation of the random sums by a Wiener or \( \alpha \)-stable Lévy process under various moment assumptions on \( \{X_i\} \) and \( \{Z_i\} \) [1]. As a consequence various modifications of the LIL and Erdős-Rényi-Csörgő-Révész-type SLLN for random sums were obtained and used for investigation the asymptotic behavior of the risk processes in classical Cramer-Lundberg and renewal Sparre Andersen risk models, the case of risk models with stochastic premiums [2] is also discussed.

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ІМОВІРНІСТЬ, НАДІЙНІСТЬ ТА СТОХАСТИЧНА ОПТИМІЗАЦІЯ

Присвячена 90-й річниці В. С. Королюка,
80-й річниці І. М. Коваленка,
75-й річниці П. С. Кнопова та
75-й річниці Ю. В. Козаченка

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